Assignment 5.

Given November 15, due November 29.

Objective: Simulate a stochastic process

We want to include default risk in pricing a simple zero coupon corporate bond. In this model, we say taht the company defaults if its stock price ever goes below $S_d = 1$. Let D be the event that $S(t) \leq S_d$ for some $t \leq T$, and let ND be the complementary event. The random variable χ will have the value 1 if ND and 0 if D. The risk adjusted present value is

$$\mathrm{E}\left[\exp\left(-\int_0^T r(t)dt\right) \cdot \chi\right] .$$

Use the following model:

$$dS = rSdt + \sigma_S S dW_1$$

$$dr = \mu(r_0 - r)dt + \sigma_r r dW_2$$

$$d\sigma_S = \nu(\sigma_0 - \sigma_S)dt + \sigma_v \sigma_S dW_3$$

with parameters

$$r_0 = 7\%$$

$$r(0) = 6\%$$

$$\sigma_S(0) = 50\%$$

$$\mu = .5$$

$$\sigma_r = 30\%$$

$$\nu = 2$$

$$\sigma_v = 80\%$$

$$\sigma_0 = 30\%$$

$$S(0) = 10$$

$$T = 10$$

Assume the correlation of W_1 and W_2 is -.3, the correlation of W_1 and W_3 is -.5, and the correlation of W_2 and W_3 is .4.

Use the forward Euler method to generate sample paths and then average the functional. Do an experiment with several fairly large values of Δt to see how the bias depends on Δt.