Mathematical modeling.

Assignment 8, due April 19.

1. The Markov chain with four states, a, b, c, and d, has transition probabilities given by the diagram:

$$
\begin{array}{ccccccc}
 & 1/4 & & 1/3 & & 1/2 & \\
 & \longrightarrow & & \longrightarrow & & \longrightarrow & \\
a & & b & & c & & d \\
 & \longleftarrow & & \longleftarrow & & \longleftarrow & \\
 & 1/3 & & 1/2 & & 1/3 &
\end{array}
$$

a. What is the transition matrix?

b. If $X(0) = a$, what is the probability that $X(4) = c$? Use Matlab to do the matrix multiplication.

c. What are the steady state probabilities? Do this by hand.

2. Suppose a company has three financial states, “healthy”, “borderline”, and “bankrupt”. If the company is healthy, it becomes borderline with probability 1/3. Otherwise it stays healthy. If it is borderline, it goes bankrupt with probability 1/3, becomes healthy with probability 1/3, and otherwise stays borderline. A bankrupt company remains bankrupt. The transition period is one year. A company can go from healthy to bankrupt, but only through two or more transitions, each one taking one year.

a. Describe this process as a Markov chain. What is the transition matrix?

b. Find the eigenvalues and eigenvectors of the transition matrix. Diagonalize the transition matrix.

c. If the company starts healthy, find the probability that it goes bankrupt after exactly t years. Be careful; a company may be bankrupt after t years because it went bankrupt earlier.

d. Find the expected time to become bankrupt. This involves knowing that

$$\sum_{n=0}^{\infty} n z^n = \frac{z}{(1-z)^2} \,.$$

3. Suppose that X and Y are independent random variables and that both are uniformly distributed in the interval $[0, 1]$. Find the PDF for $Z = x + Y$.