Optimal transport and Schrödinger bridges from a control and computational perspective

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Abstract

The dynamic version of the Monge-Kantorovich optimal mass transport (OMT)

$$\inf_{\pi\in\Pi(\mu,\nu)}\int_{\mathbb{R}^n\times\mathbb{R}^n} c(x,y)d\pi(x,y),$$

which is available whenever the cost functionn $c(x,y)$ derives from a Lagrangian action, has, in a suitable sense the Schrödinger bridge problem (SBP) as a regular approximation (Mikami 2004, Mikami-Thieullen 2006,2008, Léonard 2012, 2014). The latter, originating with Schrödinger in 1931 and with important contributions due to Fortet, Beurlin, Jamison and Föllmer, is a maximum entropy problem connected to large deviations of the empirical distribution on path-space. It has important applications, for instance, to cooling of micro and macro mechanical systems [5]. Recently, we have derived implementable forms of the solution for general linear stochastic systems [2], [3], [4]. For Markov chains and Kraus maps of statistical quantum mechanics implementable solutions have been presented in [1].

In this talk, based on joint work with Yongxin Chen and Tryphon Georgiou, University of Minnesota, I wish to unveil some profound connections between the two theories (and their generalizations) which allow, in particular, to exploit the implementable solutions of SBP to get approximate solutions of a general OMT problem.

References

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