Assignment 1: Atmospheric Dynamics

All questions require detailed explanations of answers.

1 Radiation

The surface of Venus has an approximate temperature of $750°K$. Assume that the amount of short wave radiation absorbed at the surface of Venus is the same as that of Earth (it is cloudier but closer to the Sun). Generalize the simple radiation model in Lecture 2 to obtain an estimate for the relative long wave radiative absorptivity of the Venusian atmosphere as compared to that of the Earth. Speculate on why the Earth and Venus are so different climatically given that they have nearly identical masses and radii.

2 Vorticity

How far must a zonal ring of air initially at rest with respect to the Earth's surface at $30°N$ and $50km$ height be displaced latitudinally in order to acquire an easterly velocity of $20ms^{-1}$? How far vertically must the same parcel be displaced in order to acquire the same velocity?

3 Boundary Layer

Derive an expression for the wind-driven surface Ekman layer in the ocean. Assume that the wind stress is constant and directed along the x-axis. Continuity of turbulent momentum flux at the air-sea interface ($z = 0$) requires the wind stress divided by air density must equal the oceanic turbulent momentum flux at $z = 0$. Thus if we assume that flux-gradient theory is used the boundary conditions at the surface become

$$\rho_0 K_m \frac{\partial u}{\partial z} = \tau, \qquad \rho_0 K_m \frac{\partial v}{\partial z} = 0, \qquad at\ z = 0.$$

where K_m is the eddy viscosity of the ocean (assumed constant). As a lower boundary condition assume that $u, v \to 0$ as $z \to -\infty$. Assume that $K_m = 3 \times 10^{-3} m^2 s^{-1}$ then derive the depth of the surface Ekman layer at $30°N$.

4 Quasi-geostrophic theory

Assume that the geopotential field is given by

$$\Phi = \Phi_0(p) + cf_0 \left\{ -y \left[\cos\left(\pi p / p_0\right) + 1 \right] + k^{-1} \sin k \left(x - ct\right) \right\}$$

where c is a constant speed, k a zonal wave number and $p_0 = 1000mb$.

1. Use the quasi-geostrophic vorticity equation to obtain the horizontal divergence field consistent with this geopotential under the assumption that the Coriolis parameter does not vary with latitude.

2. Assuming that $\omega(p_0) = 0$ obtain an expression for ω at an arbitrary height using the continuity (conservation of mass) equation.

3. Using a suitable software package (e.g. Matlab) plot the geopotential and divergence fields together. Indicate where positive and negative vorticity advection occurs.