CHAPTER 1

Tropical Atmosphere/Ocean Coupling and El Nino

1.1. Fundamentals of the coupling

It is well known that Sea Surface Temperature (SST) is a major control on tropical convection (thunderstorm rainfall) and hence the overall atmospheric circulation since the latent heat release associated with convection drives huge overturning cells. As the SST increases tropical convection increases since more moisture is generally available in the surface layers of the atmosphere. You can see therefore that the atmospheric circulation is likely to be sensitive to this oceanic field. We now consider what influences this variable. Generally the mixed layer near the surface of the ocean has relatively uniform properties so we may write down an equation for its temperature and this will also be the equation for SST:

$$T_t + uT_x + vT_y + \frac{w}{H_m}(T - T_b) = \frac{Q}{\rho_o C_p H_m} \tag{1.1.1}$$

where H_m is the depth of the mixed layer; T_b is the temperature of water below the mixed layer; Q is the surface heat flux into the ocean; ρ_o is the ocean density and C_p is the ocean specific heat. We note that SST is controlled by the ocean flow field (u, v, w) and by the subsurface temperature T_b as well as the heat flux from the atmosphere.

Now in general, currents near the surface of the equatorial ocean are generated by wind forcing as we saw in the under-current lecture. In addition T_b is controlled by the depth of the thermocline which again as we saw previously is strongly influenced by wind forcing. In summary then SST is strongly influenced by atmospheric forcing. This circular situation where ocean controls atmosphere via SST and atmosphere controls ocean via wind stress and heat flux implies that we need to consider both medium together in order to explain tropical climate variations. This situation is particularly true for the El Nino phenomenon. A typical event is associated with large scale warm SST anomalies such as those depicted in Figure 1.1.1.

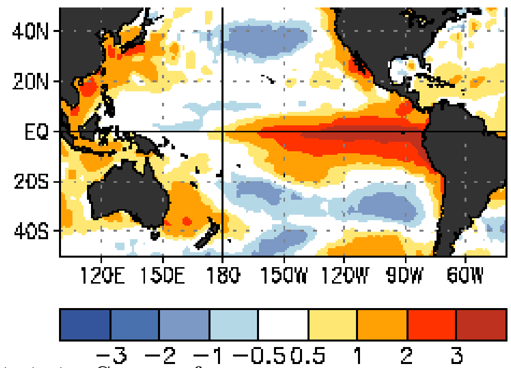

FIGURE 1.1.1. Sea surface temperature anomalies in degrees Celcius during the 1997-98 El Nino. Shown is the three month average from January 1998 through March 1998.

The typical (atmospheric) wind stress anomalies that occur are those seen in Figure 1.1.2.

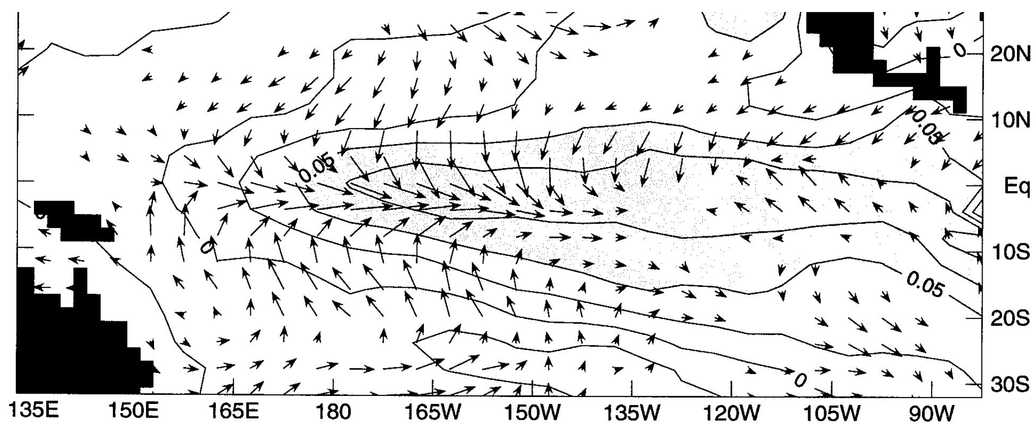

FIGURE 1.1.2. Windstress Anomalies during a strong El Nino. The maximum arrow length corresponds with a magnitude of approximately $0.1Nm^{-2}$.

Finally heat flux anomalies tend to oppose the SST change. If a good ocean computer model (such as the one we looked at in the previous Chapter) is forced by the mean wind stress for the Pacific and then the anomalies from Figure 1.1.2 are added and the model rerun then the new SST will be greater than the first runs SST by close to the amount given in Figure 1.1.1. Conversely if a good atmospheric computer model is forced firstly by the normal Pacific SST and then the anomalies from Figure 1.1.1 are added then the difference in wind stresses between the two runs will strongly resemble Figure 1.1.2. This clearly indicates that a coupled model is required to describe the dynamics of El Nino.

The atmospheric effects of El Nino have profound societal implications since they cause global circulation changes in response to the anomalous latent heating. Figure 1.1.3 summarizes the major effects. Oceanic effects are restricted to fisheries in the Americas.

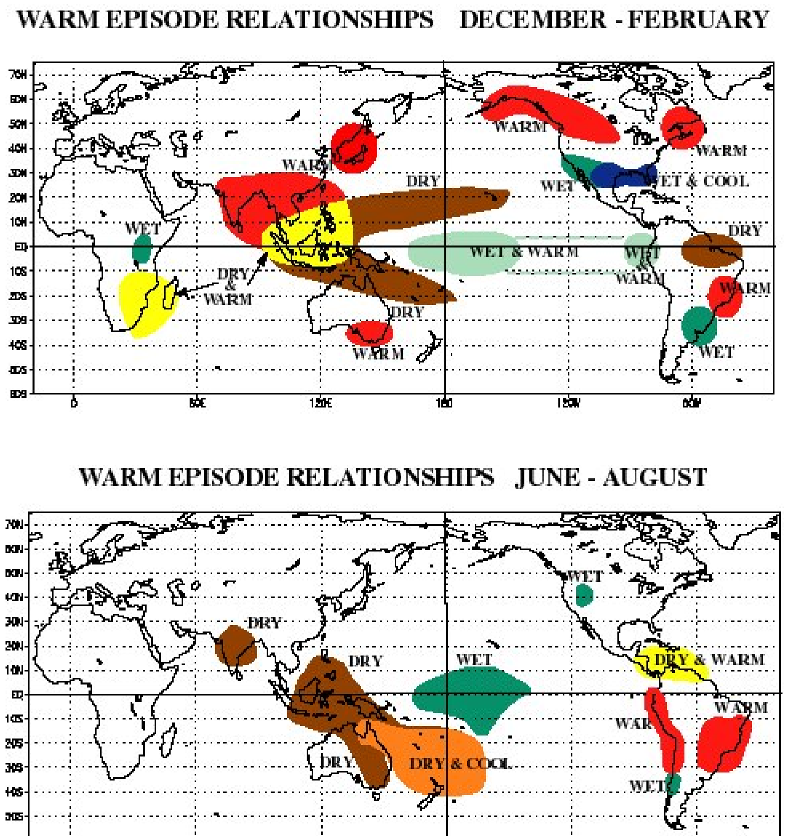

FIGURE 1.1.3. Atmospheric (and hence societal) anomalies associated with El Nino. Diagram courtesy of N.O.A.A.

It is worth analyzing in more detail the dynamical terms in the SST equation most responsible for causing changes in the equatorial Pacific. Let us linearize equation (1.1.1) about the mean state of the

equatorial Pacific:
(1.1.2)

$$T_t^{'} + \overline{u}T_x^{'} + u^{'}\overline{T}_x + v^{'}\overline{T}_y + \overline{v}T_y^{'} + \frac{\overline{w}}{H_m}(T^{'} - T_b^{'}) + \frac{w^{'}}{H_m}(\overline{T - T_b}) = \frac{Q^{'}}{\rho_o C_p H_m}$$

On the equator where coupling originates, a number of simplifications are possible. Firstly meridional currents and temperature gradients are small; secondly the mean zonal current in the mixed layer tends to be small as a result of the large vertical shear there (remember undercurrent model results from the previous lecture); thirdly variations in $T_b^{'}$ are strongly related to the anomalous thermocline position h; finally heat flux tends overwhelmingly to be a negative feedback. Thus our linearized equation may be reduced to the approximate form

$$T_t^{'} + u^{'}\overline{T}_x + \frac{\overline{w}}{H_m}(T^{'} - rh) + \frac{w^{'}}{H_m}(\overline{T - T_b}) = -sT^{'} \tag{1.1.3}$$

We see therefore that the major controls on variations in equatorial SST are

(1) Zonal current anomalies. Transient zonal currents with uniform vertical structure in the mixed layer can be large as you may have noted in the undercurrent model. In addition there is a strong east west gradient in Pacific mean SST so the second term in equation (1.1.3) may be large.
(2) Thermocline depth variations. We saw previously that there is a large east west gradient in the climatological thermocline depth which is a consequence of the trade-winds. Changes in trade winds cause changes in this i.e. result in non-zero h. Note however that the mean equatorial upwelling $\overline{w}$ is required to bring this signal into the mixed layer. Thus the effects of thermocline variations tends to be greatest in the eastern Pacific where mean upwelling is largest.
(3) Upwelling anomalies. The equatorial upwelling is a result of surface Ekman divergence at the equator and that this is caused directly by the trade winds and their strong easterly zonal windstress (see first question on previous assignment). Changes in this stress can obviously drive changes in upwelling. Note that for upwelling anomalies to be important we require that the mean temperature difference between mixed layer and subsurface be large. This generally only occurs in the eastern Pacific where windstress changes during El Nino are not strong (see Figure 1.1.2 above). Thus this term tends to be less important than the first two. This reduced importance is less

true for the seasonal cycle however where significant changes in far eastern Pacific zonal windstress are more important.

1.2. Coupled Instabilities

When the atmosphere and ocean are coupled together the combined system has the potential for linear instability and hence rapid growth of climate system anomalies such as SST. We consider a linear system of equations describing this situation and solve these with certain approximations. In this section since we are considering linear instability we drop primes on anomalous fields.

A revealing simple model was proposed by Neelin (around 1990) in order to explore the growth rate and propagation direction of instabilities. Here the atmospheric model is reduced considerably: It is assumed that positive zonal wind anomalies occur to the west of the heating and negative anomalies occur to the east. A useful way of viewing this is to say that there is a zonal phase shift between the heating and the zonal wind. Motivated by this we introduce the atmospheric model for the (complex) Fourier component with respect to the zonal coordinate x of zonal wind U

$$U = e^{i\phi} Q = \alpha e^{i\phi} T \tag{1.2.1}$$

where ϕ is a phase shift angle (typically taken to be around $\pi/4$) and we also assume that heating is proportional (via the positive coefficient α) to SST anomaly T. Note that Q and T are also taken to be Fourier components.

Two further simplifying assumptions can be made by assuming that the thermocline perturbations are in approximate Sverdrup balance and that the zonal current anomalies are approximately proportional to the windstress anomalies (see undercurrent lecture notes to work out for yourself the validity of these simplifying approximations). Neelin refers to this steady state assumption as the "fast wave limit" as the slow adjustment of the ocean has been eliminated and while its limitations are obvious, the simplification is fairly reasonable under the circumstances of a growing disturbance with timescales of a month or two. Thus we write

$$\begin{aligned} h_x &= cU \\ u &= dU \end{aligned} \tag{1.2.2}$$

where c and d can be checked to be positive. We can write down the equatorial SST equations in simplified form as

$$T_t + u\overline{T}_x - eh = -rT$$

where we are taking into account the two main processes controlling equatorial SST and we are combining two forms of decay of anomalies into the right hand side. The effects of equatorial upwelling anomalies could easily also be added since such anomalies are proportional to zonal windstress anomalies in the same way as zonal currents are assumed to be in the second equation of (1.2.2). If we now assume as usual that the Fourier components have the form

$$\begin{aligned} T &= T_0 e^{i(kx-\omega t)} \\ U &= U_0 e^{i(kx-\omega t)} \\ u &= u_0 e^{i(kx-\omega t)} \\ h &= h_0 e^{i(kx-\omega t)} \end{aligned}$$

then we obtain firstly that

$$h = \frac{-i}{k} cU$$

and hence that

$$\begin{aligned} (-i\omega + r)T &= -\overline{T}_x dU - \frac{i}{k} ceU \\ &= e^{i\phi}\alpha \left(-d\overline{T}_x - \frac{ice}{k} \right) T \end{aligned}$$

hence upon expansion we obtain

$$i\omega = (\sin\phi + i\cos\phi)\left(\alpha d\overline{T}_x + \frac{ic\alpha e}{k}\right) + r$$

and so

$$\begin{aligned} \omega_r &= \alpha d\overline{T}_x \cos\phi + \frac{c\alpha e}{k}\sin\phi \\ \omega_i &= \frac{c\alpha e}{k}\cos\phi - d\alpha\overline{T}_x \sin\phi - r \end{aligned} \tag{1.2.3}$$

Now all the coefficients are positive except for $\overline{T}_x$ which is negative for the Pacific so therefore according to the first equation of (1.2.3) the zonal advection term of the SST equation induces westward propagation[1] while the thermocline term induces eastward propagation. The second equation shows that both terms cause instability. Of course this is all under the assumption that the phase ϕ is between 0 and $\pi/2$.

[1]It is easily checked that positive values for ω_r occur when a Fourier component corresponds with a wave moving in a positive i.e. eastward direction. Likewise positive values of ω_i are easily checked to correspond with exponential growth while negative values imply exponential decay.

These mechanisms are easily understood conceptually as follows. Let us assume that a small sinusoidal windstress anomaly occurs. This will generate a sinusoidal anomaly in h which is phase shifted $\pi/2$ to the east because of the Sverdrup balance assumption. The h will induce an SST anomaly with the same zonal phasing as itself. The SST anomaly will then induce a zonal wind stress anomaly phase shifted ϕ *to the west.* If the phase shift is $\pi/4$ then the resulting additional windstress anomaly will act to reinforce the original anomaly and also to shift it to the east. Similar comments apply to explaining the westward growing disturbance associated with zonal advection.

All the above coupled instability mechanisms ignore the slow ocean adjustment process mediated by Rossby and Kelvin waves that we examined in the undercurrent Lecture. In models of El Nino able to successfully predict the phenomenon it turns out that this slow process is fundamental. Despite this limitation, the above analysis explains well the *rapid* instability properties of the coupled system. It may also be extended to cover the case of Ekman upwelling anomalies which behaves the same way as the zonal advection anomaly case.

1.3. The dynamic character of El Nino

This phenomenon is by far the largest form of variability in the climate system for frequencies less than 100 years. The basic structure of SST and zonal wind stress variations are depicted in Figure 1.3.1 which shows anomalies for the past decade or two. These were collected from a large observational array of oceanic buoys located within 300km of the equator right across the Pacific. The Figure shows an equatorial time section of the variables and a number of things are apparent.

(1) As was noted above the SST anomalies occur in the eastern half of the basin where the mean position of the thermocline is shallow.
(2) The zonal wind stress anomalies occur with a significant westerly displacement relative to the SST anomalies. Convection anomalies (not shown) also have this westerly displacement but to a lesser extent. As we noted above this displacement is important for the development of coupled instabilities.
(3) There is little or no evidence of significant propagation in the SST anomalies which grow in a stationary pattern right across the eastern part of the basin.
(4) The fluctuations are irregular but a time scale of roughly four years may be discerned.

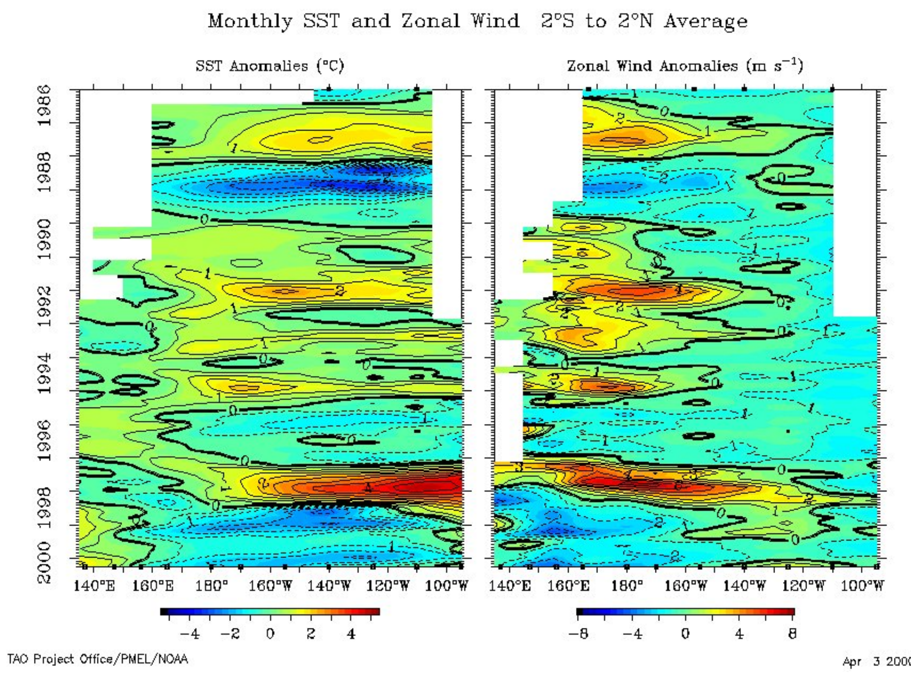

FIGURE 1.3.1. Equatorial Time-Longitude plots of SST and Zonal Wind anomalies for years 1986-2000. Courtesy of the Pacific Marine Environment Laboratory.

An adequate theory of ENSO must be able to explain convincingly all four of these fundamental features. We have seen above how the first two features are explicable in terms of atmospheric and oceanic dynamics. The last two have been clarified in the past 5 years although some controversy still remains concerning the irregularity of ENSO.

Consider now the implications of the third feature of ENSO: We saw in the previous Lecture that plausible instability mechanisms exist which can explain the growth of anomalies. Eastward propagation is associated with thermocline processes while westward propagation is connected with zonal current processes. One could theoretically eliminate propagation by combining these two effects however no convincing model without boundaries has been able to reproduce satisfactorily this feature of the observations. Realistic models with this feature and the ability to predict ENSO require zonal boundaries. We consider now the first such model which was introduced some 25 years ago by Mark Cane and Steve Zebiak from Columbia University.

1.4. A "simple" but realistic model of ENSO

The Cane and Zebiak model consists of a depiction of the tropical Pacific basin (taken to be rectangular). It is an anomaly model in that it depicts deviations from mean (seasonally varying) variables and has three components which we now discuss in detail

1.4.1. Ocean dynamics. The upper $H = 150m$ of the tropical Pacific is divided into two layer of mean depths $100m$ and $50m$ with the latter layer taken to be the mixed layer. The total flow u_H in the two layers is assumed to obey shallow water equations for the first baroclinic mode on an equatorial β plane:

$$\begin{aligned} (u_H)_t + au_H - \beta y v_H &= -h_x + X/\rho_o H \\ (v_H)_t + av_H + \beta y u_H &= -h_y + Y/\rho_o H \\ h_t + ah + c^2 \nabla \cdot \overrightarrow{u}_H &= 0 \end{aligned} \tag{1.4.1}$$

This is a fairly good approximation for this flow as the first baroclinic mode is the major mode stimulated by wind stress in the central Pacific and the anomalous flow in the first $150m$ is controlled (in a linear sense) by the first three or so baroclinic modes. The linear damping a is chosen to have a time scale of around 3 years which is of the same order that the McCreary undercurrent model discussed previously had for the first vertical mode.

The difference in flow between the mixed layer and the lower layer is controlled by the shear flow which as we saw in the undercurrent model is mainly due to high-order baroclinic modes. These are heavily damped and so Cane and Zebiak assumed that all these modes could be combined into a simple heavily dissipated and steady state equation for the shear or Ekman flow:

$$\begin{aligned} \epsilon u_S - \beta y v_S &= X/\rho_o H_m \\ \epsilon v_S + \beta y u_S &= Y/\rho_o H_m \end{aligned} \tag{1.4.2}$$

where the damping ϵ is assumed to be of order several days and $H_m = 50m$ is the mixed layer depth. In some loose sense then the ocean component of the model of Cane and Zebiak is a simplified version of McCreary's linear model of equatorial dynamics. The simplification results in degradation in the depiction of zonal currents but thermocline displacements (h) are still done quite well.

1.4.2. SST equation. Above we considered the temperature equation for the ocean mixed layer as this is essentially an equation for SST. Cane and Zebiak used the perturbation form of this equation (i.e. perturbed about the mean state of the tropical Pacific) but assumed that the mixed layer has a constant depth of $50m$. They also introduced a fairly complicated parametrization of T_b' (the subsurface temperature anomaly) in terms of the thermocline displacement anomaly h. This parametrization has a number of features which we briefly describe

(1) T_b' varies directly and monotonically with h. For "moderate" values of h the relationship is approximately linear.
(2) The relationship is strongly dependent on zonal location. In the eastern Pacific the relationship is much stronger than in the central Pacific.
(3) The relationship has an "amplitude limiting" non-linearity. Thus if the magnitude of h becomes large enough then T_b' approaches a limiting value.

These features can be justified physically and cause the model to oscillate realistically.

All the anomalous currents and h required for calculation of the SST anomaly are obtained from equations (1.4.1) and (1.4.2).

1.4.3. Atmospheric component. The diabatic heating is given by a crude convection parametrization but for all practical purposes it is approximately proportional to the SST anomaly. The windstress response can be modeled quite simply using steady damped linear shallow water equations and solutions resemble the observations depicted in Figure 1.1.2 quite well.

1.4.4. Model behavior. The model is integrated numerically and develops oscillations which are depicted in Figure 1.4.1. These oscillations show a number of strong similarities with the observations seen in Figure 1.1.1: The SST anomalies are standing in character and are confined to the eastern part of the basin; the wind stress anomalies are displaced to the west of the SST anomalies; and finally the period of the oscillation is around 3-4 years which is where the spectral peak is seen in observed data.. There are a number of disagreements still with data with the principal one being that oscillations are located too far to the east in the model. Nevertheless the agreement with data is remarkable given the relative simplicity of the model described above. The nature of the model behavior has therefore attracted much analysis (see below) and has been reproduced in a wide range of different models since.

1.5. The delayed action oscillator paradigm

The model of Cane and Zebiak was analyzed in much detail by Battisti and Hirst in the late 1980s. At the same time a somewhat more complex model due to Schopf and Suarez was similarly analyzed (by that models

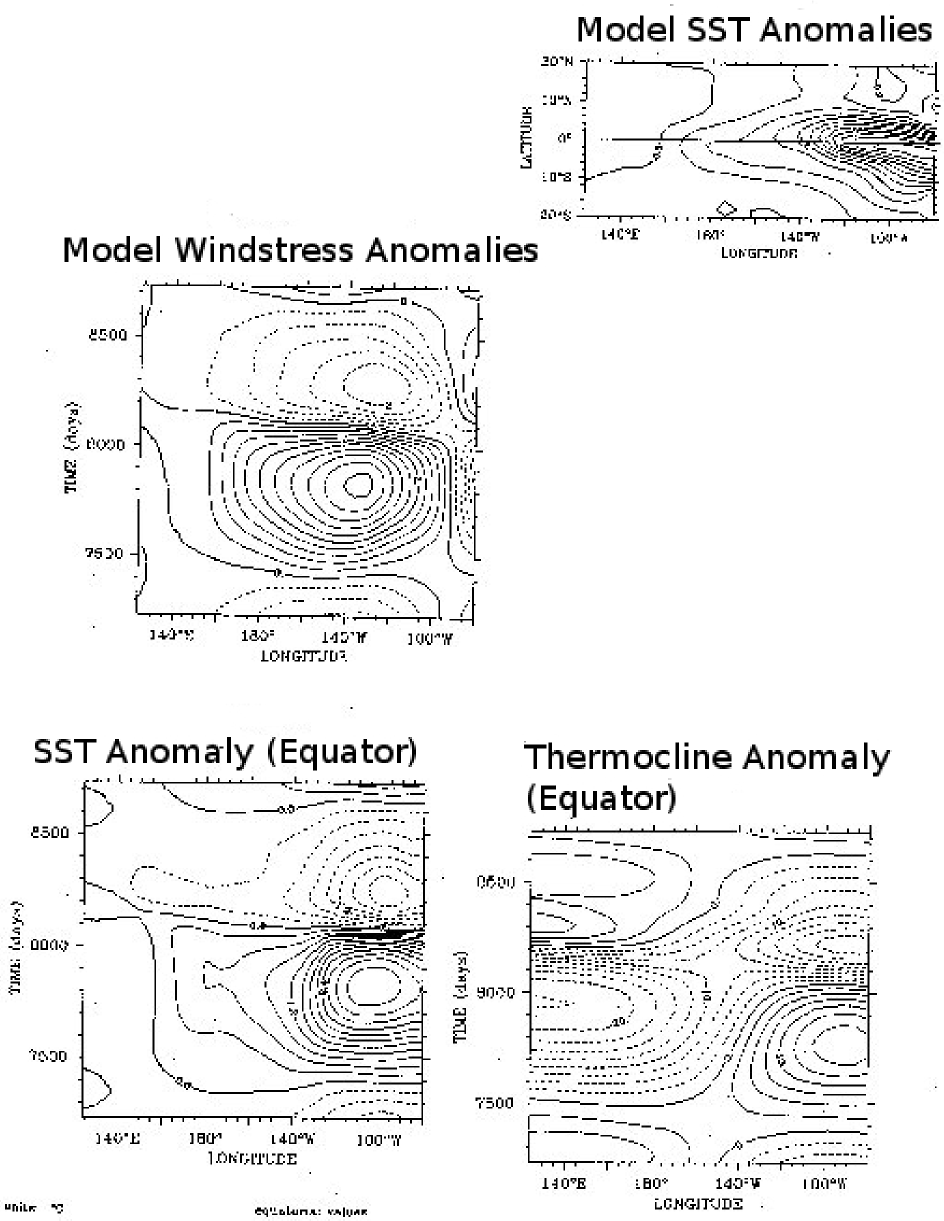

FIGURE 1.4.1. Cane and Zebiak model oscillatory behavior. Depicted are various dynamical fields and their evolution. A snapshot of *SST* anomaly is also included during an El Nino.

authors) with the same basic conclusion as to mechanisms. The Schopf and Suarez model exhibited a very similar oscillation to that seen in the Cane and Zebiak model. We concentrate our discussion here on the analysis due to Battisti and Hirst.

By means of a series of careful sensitivity experiments a number of things were discovered concerning the oscillations seen in Figure 1.4.1. These included

(1) The rapid growth of disturbances in the eastern Pacific was a result of the three coupled instability mechanisms discussed earlier i.e. it was due to zonal advection; thermocline perturbation and upwelling anomaly processes in the SST equation.
(2) The western boundary condition of the model was critical. Without it no oscillation occurred.

Why should the western boundary in particular be important?

Consider the effects on the ocean of an increasing westerly zonal wind anomaly in the central Pacific: This anomaly will generate a positive Kelvin wave which will propagate to the east causing the coupled instability noted previously. It will then reflect from the eastern boundary as *positive* Rossby waves which will tend if anything to reinforce the instability growth taking place in the eastern part of the basin. At the same time as this process is taking place another is taking place in the western part of the basin. Here the westerly wind anomalies generate a packet of *negative* Rossby waves. These do not affect SST all that much *immediately* because there is little mean upwelling in the western part of the Pacific. After some time interval, determined by the distance between the wind anomalies and the western boundary as well as the Rossby wave speed, the Rossby waves reach the western boundary and are reflected as a *negative* Kelvin wave. This wave moves rapidly to the eastern part of the basin where most of the action is taking place and acts as a powerful negative feedback on the coupled instability there. This negative feedback, which occurs with some time delay, therefore has the potential to terminate or even reverse the large SST anomalies occurring in the eastern Pacific. This was indeed the process occurring in the Cane and Zebiak oscillations.

Since most of the variability in particular climate variables is taking place in fixed spatial locations (the oscillation is standing) we can legitimately simplify the Cane and Zebiak model oscillation into the following useful "toy" model describing area averaged eastern equatorial Pacific SST anomaly:

$$T(t)_t = cT(t) - bT(t-\tau) \tag{1.5.1}$$

Here the coefficients c and b are assumed positive and τ is the time delay involved in propagation of the Rossby waves to the western boundary and there subsequent return as a Kelvin wave. We are ignoring amplitude limiting non-linearities and the eastern boundary reflections for simplicity as these do not change the qualitative conclusions. The reason we use a term proportional to $T(t-\tau)$ to describe the negative feedback is because the wind stress which forced the Rossby waves at this earlier time was of course proportional to the SST anomaly at that time.

Equation (1.5.1) is known as a delay differential equation and is soluble in principle. If we choose solutions of the form

$$T = T_0 \exp(\sigma t)$$

where σ is complex then substitution into (1.5.1) results in the complex equation

$$\sigma = -b \exp(-\sigma\tau) + c \tag{1.5.2}$$

Solutions for this are readily found by numerical solution (or analytically using the Lambert W function) and have the following properties: When

$$0 < b < \exp(c\tau)/\tau$$

only exponentially growing solutions occur (with reduced growth rate compared to the case $b = 0$). When $b > \exp(c\tau)/\tau$ oscillatory solutions become possible either damped or growing. This shows that the delayed negative feedback must be sufficiently strong for oscillatory solutions to occur, a rather intuitively obvious situation. The situation is displayed graphically in Figure 1.5.1 which shows the complex dispersion relation corresponding to solutions of (1.5.2). The differing curves are for different values of b while the shaded areas are those corresponding to best estimates of b and c for the Cane and Zebiak model. The x-axis is the coupled instability parameter c which clearly influences both the growth rate (intuitively obvious) and the period of the oscillation (less obvious). The delay time for this diagram is chosen to be half a year. The conclusion of this analysis is that oscillatory solutions are possible for a broad range of model parameters and that the period of the oscillation is set in a rather complex fashion by the coupled instability timescale; the delayed negative feedback timescale and also the reflection efficiency from the western boundary. In the Cane and Zebiak model which has a self-sustaining oscillation, clearly the growing linear oscillatory solution is appropriate however the amplitude limiting non-linearity on the thermocline term discussed previously ensures a finite amplitude weakly non-linear solution.

$$\frac{\partial T}{\partial t} = -b\, T(t-\tau) + c\, T$$

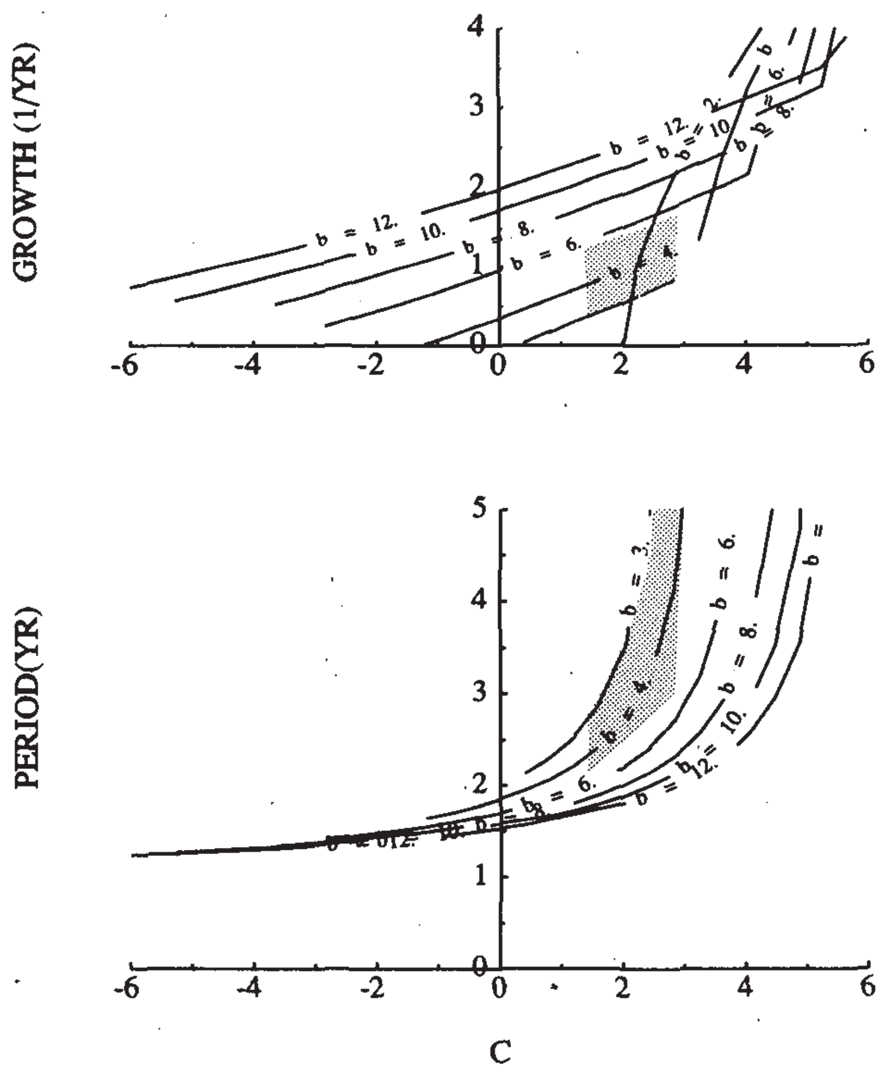

Figure 1.5.1. Complex dispersion relations for the delayed differential equation underlying El Nino. The real and imaginary parts of the frequency correspond to the period and growth rate of solutions.

More complex and realistic modeling done since Cane and Zebiak has confirmed that the so-called "delayed action oscillator" mode of coupled variability is a very robust feature of coupled models and it is now widely believed by most climate scientists that the paradigm outlined above describes the basic dynamics of ENSO.

1.6. Model Description

We use a coupled ocean atmosphere model which is conceptually similar to the Cane and Zebiak model detailed above. It was developed in the early 1990s by the author to study the nature and dynamics underlying El Nino predictability. Complete documentation can be found in Kleeman (1991) and Kleeman (1993). We provide a brief summary here on the ways in which it differs from the Cane and Zebiak (CZ) model.

The basic ocean dynamics are essentially identical to CZ in that equations (1.4.1) and (1.4.2) are used to describe the currents within the upper mixed layer of the ocean which is assumed like CZ to have a constant depth of $50m$. The temperature equation for the mixed layer is a significant simplification of the CZ version. Equation (1.1.2) is used on the equator and off-equatorial temperature anomalies are simply assumed to be proportional to equatorial temperatures with a constant of proportionality which drops according to a Gaussian profile away from the equator. This simplification is valid because in reality SST anomalies are mainly created on the equator and transmitted north and south fairly rapidly by meridional advection. The equatorial equation (1.1.2) is simplified by dropping several small advection terms and simplifying the vertical advection and heat flux terms. The resulting equatorial SST anomaly equation reads

$$T'_t = f(x)M(h') - \kappa T'$$

where in order to ensure finite amplitude behaviour we set

$$\begin{aligned} M(h') &= h' & |h'| &< h_{max} \\ &= h_{max} & h' &> h_{max} \\ &= -h_{max} & h' &< -h_{max} \end{aligned}$$

This "cutoff" function reflects the reality that if the thermocline gets too deep, further deepening has little further effect on SST. Similarly if the thermocline surfaces (i.e. h' is very negative) little further cooling influence can be expected for more negative thermocline anomalies. Thus SST anomalies never really exceed an amplitude of around 4 or 5 degrees.

The function $f(x)$ increases from west to east and reflects the greater sensitivity of SST to thermocline movements in the eastern rather than western Pacific. This occurs partly because the upwelling is much greater in the former region.

In some rough sense this ocean model of SST anomaly represents a minimal realistic simulation of equatorial variations.

The atmospheric model incorporates a simplified steady state moist convection formulation which forces a Gill (1980) linear damped shallow water set of equations. The difference from the CZ model is that due to a different convection formulation, the atmosphere responds much more strongly to SST anomalies in regions of high mean SST (mainly the western Pacific) rather than low mean SST (the eastern Pacific).

This model coupled model has been used extensively to predict El Nino and exhibits a level of skill comparable with those exhibited by more complex models used today for routine forecasts.

As was discussed above any model explanation of El Nino must also explain the irregularity of El Nino as well as its spectral peak which occurs at roughly 4 years. One explanation for this is that large scale tropical weather patterns are able to disrupt regular oscillations. Such a disruption is inherently a random process since on climate time scales weather acts like noise. This effect has been incorporated into the present model by forcing the model with large scale patterns of heat flux and wind stress which resemble certain weather phenomenon (e.g. the Madden Julian Oscillation). With this inclusion the present model exhibits quite realistic variability as documented in Moore and Kleeman (1999).

1.7. Model Interface

As in previous chapters a Matlab graphical user interface is provided for easy model operation. An example of this interface is shown in Figure 1.7.1

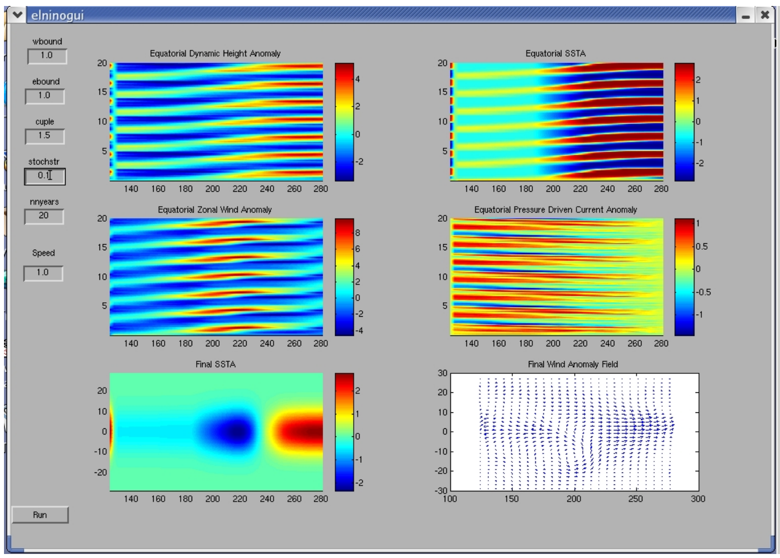

FIGURE 1.7.1. Matlab graphical interface for the El Nino model

As in previous chapters **ensure that all parameter values are reset before running the model.** The model is initialized by a constant equatorial SST anomaly of $1°C$ and may be integrated inexpensively for many decades. The model parameters are explained in Table 1

The panels on the right of the interface have the following significance:

Top left: Longitude-time plot of equatorial thermocline (dynamic height) anomaly.

Top right: Longitude-time plot of equatorial SST anomaly.

Center left: Longitude-time plot of equatorial zonal wind anomaly.

Center right: Longitude-time plot of equatorial pressure driven zonal currents.

Bottom left: Longitude-Latitude plot of SST anomaly at the end of the integration.

Bottom right: Longitude-Latitude plot of wind anomaly at the end of the integration.

Model Parameter	Meaning
wbound	Western boundary reflection efficiency coefficient. A value of 1.0 means perfect reflection of incoming Rossby waves into Kelvin waves. Values less than unity imply some absorption or transmission of the Rossby waves.
ebound	Eastern boundary reflection efficiency coefficient. Same as wbound but the reflection coefficient of Kelvin waves into Rossby waves at the eastern boundary.
cuple	Atmosphere Ocean coupling coefficient. The greater this parameter the greater the feedback between the media. A value of unity corresponds approximately with observed conditions in the Pacific.
stochstr	Stochastic forcing with spatial patterns of the large scale tropical weather have been added to the model. A value of 1.0-3.0 corresponds approximately with observations.
nnyears	Number of years to integrate model forward.
speed	Factor to multiply the shallow water/Kelvin wave speed. A value of 1.0 corresponds to a speed of 2.3 m/s.

TABLE 1. El Nino model parameter meanings.

1.8. Model Exercises

(1) (Theory question) Derive the complex dispersion relation for the delayed action oscillator model considered above i.e. equation (1.5.1). By using the following substitutions:

$$\begin{aligned} b &\equiv \varepsilon exp(c\tau)/\tau > 0 \\ z &\equiv (c-\sigma)\tau \end{aligned}$$

write these in a particularly simple form involving only ε and z. After further (obvious) manipulation use the matlab zero finding facility (or any method you desire) to find solutions as a function of ε. This is a potentially challenging exercise and full marks will be given for good partial attempts.

(2) (Model question) In the gui choose solutions with the stochastic forcing set to zero and the coupling strength at 1.5. How does the period of the oscillation vary with shallow water speed? By varying the coupling strength study the effect of the shallow water speed on the model stability.

(3) (Model question) Repeat question 2 but study the effect of the western boundary reflection coefficient. Are these relations consistent with the dispersion relations derived in class and displayed in Figure 1.5.1? Provide a convincing argument for your belief.

(4) (Model question) By varying the stochastic forcing and the coupling strength derive some conclusions about when a self-sustaining oscillation is possible in this model. Hint: the input of stochastic forcing acts as an energy source for the system. Compare the solutions critically with those of Figure 1.3.1 from the Lecture notes.

(5) (Model question) What effect does the eastern boundary have on solutions?

(6) (Model question) Study the (sometimes large) pressure driven currents in the solutions. Speculate on why they are apparently only in the western part of the basin. Why do they show so much high frequency variability (for example, being large and positive right at the start of a warm event and weak and negative towards the end)?