Derivative Securities – Notes and errata, 12/5/00

(1) Reminder about the final exam:

- The exam will be Tuesday December 19, in the usual class time and place.
- I will distribute a short study guide (a list of topics to review) in the last lecture, Tuesday December 12. Most (possibly all) of the December 12 class will be devoted to answering questions and reviewing what we've covered.
- The exam will be closed-book, however you may bring two pages of notes (8.5×11, both sides, any font). Calculators are permitted but are not likely to be of use.
- The exam questions will cover fundamental ideas and examples covered in the lectures and homeworks.

(2) Comment about Problem 3 on Homework 6: this is a straightforward application of Black's model, however the data you need is somewhat hidden in the financial jargon (sorry). See page 4 of the Section 11 notes for clarification.

(3) Erratum from Sections 8 and 9: in the version of Section 8 handed out in class, the discussion of Black's formula (page 8), I wrote "The value of an option with payoff $f(s(T))$ is evidently $e^{-rT}E[f(F_0e^Z)]$ where Z has mean zero and variance σ^2T." That's wrong – the mean of Z is $-\frac{1}{2}\sigma^2$. The same error occurred on page 5 of Section 9 (discussion of Black's formula again). The formulas given for calls and puts are correct however. One way to understand why Z has mean $-\frac{1}{2}\sigma^2T$: recall that in our original derivation of the Black-Scholes formula, $E_{\mathrm{RN}}[f(s(T)] = E[s_0e^X]$ where X is Gaussian with mean $(r-\frac{1}{2}\sigma^2)T$ and variance σ^2T; for Black's formula the risk-neutral probability is the one associated with $r=0$. A second, equivalent way to understand it: the futures price solves $dF = \sigma F dw$ with respect to the risk-neutral probability, since F is a martingale. So $d(\log F) = F^{-1}dF - \frac{1}{2}F^{-2}dFdF = \sigma dw - \frac{1}{2}\sigma^2 dt$ by Ito's formula. Thus $\log F(T)$ is Gaussian with mean $-\frac{1}{2}\sigma^2T$ and variance σ^2T.

(4) Errata from Section 10: The example at the bottom of page 2 has the wrong numbers for the period when t is six months and T is one year; the displayed formula for this period should read

$$B(t,T)(.028+1) = \frac{1}{1+.028}(.028+1) = 1.$$