Derivative Securities – Homework 2 – due 10/3/00
Solutions will be distributed 10/10/00

(1) Consider the one-period trinomial model with

- asset 1 = risk-free, interest rate $r > 0$
- asset 2 = risky, initial unit price s_0, final unit prices $s_0 d$, s_0, $s_0 u$

with $d < 1 < u$. Assume that $d < e^{rT} < u$ so the market admits no arbitrage. You want to buy a call option on the risky asset with strike price K. What are the largest and smallest prices you should consider paying for it, based on considerations of arbitrage?

(2) Consider a forward contract on a non-dividend-paying stock, with strike price K and maturity T. Its value at time 0 is $s_0 - Ke^{-rT}$, where r is the risk-free rate (assumed constant) and s_0 is the stock price at time 0. We explained this in Section 1, using the standard "cash-and-carry" argument. Explain how that argument can be formalized using a one-period model with two assets and M states.

(3) Consider the following one-period market with 3 assets and 4 states:

- Asset 1 is a riskless bond, paying no interest.
- Asset 2 is a stock with initial price 1 dollar/share; its possible final prices are d and u, with $d < 1 < u$.
- Asset 3 is another stock with initial price 1 dollar/share and possible final prices d and u (same d and u).
- To keep the arithmetic simple, let's assume that $u = 1 + \epsilon$ and $d = 1 - \epsilon$ for some $\epsilon > 0$. To avoid confusion, let's number the states: 1 = both stocks go up; 2 = asset 2 goes up, asset 3 goes down; 3 = asset 2 goes down, asset 3 goes up; 4 = both stocks go down.

(a) What is the associated cash-flow matrix $D_{i\alpha}$?

(b) Find all the risk-neutral probabilities.

(c) Consider the contingent claim with payoff $f = (f_1, f_2, f_3, f_4)$. What are the smallest and largest prices for f permitted by arbitrage considerations? (Let's call these $V_-(f)$ and $V_+(f)$.)

(d) Does $f_\alpha \geq 0$ for all α and $V_-(f) = 0$ imply $f = 0$? Explain.

(e) Which f's are replicatable?

(4) It is said that a London betmaker gave the following odds on the 1996 US Presidential election: 6-1 in favor of Clinton, 7-2 against Dole, 50-1 against Perot. Interpret this to mean that the betmaker was willing to take only three types of bets – that Clinton would win, that Dole would win, and that Perot would win – and

- 6 dollars bet on Clinton would return 7 if he won, 0 if not;
- 2 dollars bet on Dole would return 9 if he won, 0 if not;
- 1 dollar bet on Perot would return 51 if he won, 0 if not.

Interpret this as a one-period market with three assets: a 1-dollar bet on Clinton, a 1-dollar bet on Dole, and a 1-dollar bet on Perot. What are the associated risk-neutral probabilities? How much was the betmaker taking of every dollar bet? Explain. (This problem is adapted from Marco Avellaneda's notes.)