Derivative Securities – Homework 6 – distributed 11/28/00, due 12/12/00 – NO GRACE PERIOD
Solutions will be distributed 12/12/00

1) [Jarrow-Turnbull chapter 14, problems 1 and 2, somewhat modified.] Suppose the LIBOR discount rate $B(0,t)$ are given by the table below. Consider a 3-year swap whose floating payments are at the then-current LIBOR rate, and whose fixed payments are at the term rate of R_{fix} per annum.

payment date t_i	$B(0,t_i)$
0.5	.9748
1.0	.9492
1.5	.9227
2.0	.8960
2.5	.8687
3.0	.8413

(a) Suppose R_{fix} is 6.5 percent per annum and the notional principal is 1 million dollars. What is the value of the swap?

(b) What is the par swap rate? In other words: what value of R_{fix} sets the value of the swap to 0?

2) In the Section 10 notes we found the value of a swap by considering the two associated bonds. An alternative viewpoint views the swap as a collection of forward rate agreements. Let's verify that this gives the same result.

(a) Show that the value at time 0 of the forward rate agreement associated with payments at time t_i is

$$B(0,t_i)[R_{\text{fix}} - f_0(t_{i-1},t_i)](t_i - t_{i-1})L$$

where L is the notional principal and

$$f_0(t_{i-1},t_i)(t_i - t_{i-1}) = \frac{B(0,t_{i-1})}{B(0,t_i)} - 1.$$

(b) Suppose the payment dates are $0 < t_1 < \ldots < t_N$. Show that adding the values obtained in (a) gives the total

$$\sum_{i=1}^{N} B(0,t_i)R_{\text{fix}}(t_i - t_{i-1})L - (1 - B(0,t_N))L.$$

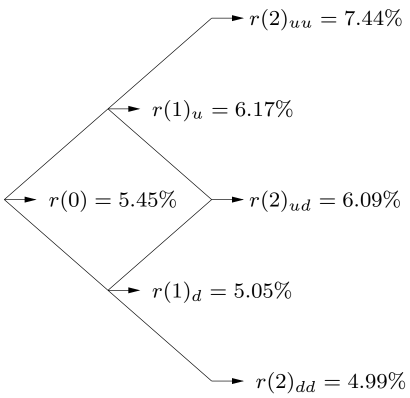
(This is the formula implicit in the Section 10 notes, obtained by comparing the two bonds; see especially the example on page 5.)

3) [Hull, Chapter 20, problem 22.] Calculate the price of a cap on the three-month LIBOR rate in nine months' time when the principal amount is $1000. Use Black's model and the following information:

- The nine-month Eurodollar futures price is 92 (ignore the difference between forwards and futures).
- The interest rate volatility implied by a nine-month Eurodollar option is 15 percent per annum.
- The current 12-month interest rate with continuous compounding is 7.5 percent per annum.
- The cap rate is 8 percent per annum.

4) [Hull, Chapter 20, problem 23] Suppose the LIBOR yield curve is flat at 8% with annual compounding. Consider a swaption that gives its holder the right to receive 7.6% in a five-year swap starting in four years. Payments are made annually. The volatility for the swap rate is 25% per annum and the principal is $1 million. Use Black's model to price the swaption.

5) [Jarrow & Turnbull, Chapter 15, problem 4.] Consider the binomial tree of interest rates shown in the figure (each time interval is one year, and the rates shown are per annum with continuous compounding). Assume the risk-neutral probabilities are 1/2 for each branch.



(a) Find the values of $B(0,1)$, $B(0,2)$, and $B(0,3)$.

(b) Consider the following European call option written on a one year Treasury bill: its maturity is $T = 2$, and its strike is 0.945, so the payoff at time 2 is $(B(2,3) - 0.945)_+$. Find the value of this option at time 0.

(c) Suppose you wish to hedge this option using two-year and three-year treasury bills. Find the hedge portfolio at time 0.

(Comment: this problem goes slightly beyond the material covered in lecture and in the Section 9 notes. See Section 15.3 of Jarrow-Turnbull for discussion of pricing and hedging options on bonds using an interest rate tree.)