Derivative Securities – Homework 6 (complete version) – distributed 12/6/04, due 12/13/04

Note 1: As previously announced, the final exam will be Monday December 20, in the normal class hour and location. You may bring two pages of notes (8.5 × 11, both sides, any font). The exam questions will focus on fundamental ideas and examples covered in the lectures and homework.

Note 2: The "first installment" of HW6 posted 12/2/04 had just 4 problems, corresponding to material covered in lecture on 11/29/04. This "complete version" consists of those 4 problems plus two more on material covered 12/6/04.

1) [Jarrow-Turnbull chapter 14, problems 1 and 2, somewhat modified.] Suppose the LIBOR discount rate $B(0,t)$ are given by the table below. Consider a 3-year swap whose floating payments are at the then-current LIBOR rate, and whose fixed payments are at the term rate of R_{fix} per annum.

payment date t_i	$B(0,t_i)$
0.5	.9748
1.0	.9492
1.5	.9227
2.0	.8960
2.5	.8687
3.0	.8413

(a) Suppose R_{fix} is 6.5 percent per annum and the notional principal is 1 million dollars. What is the value of the swap?

(b) What is the par swap rate? In other words: what value of R_{fix} sets the value of the swap to 0?

2) There are two ways to think about the value of a swap:

(i) One approach (presented in class on 11/29/04) views the swap as a collection of forward rate agreements. If the payment dates are $0 < t_1 < \ldots < t_N$ and L is the notional principal, this approach gives

$$\text{swap value} = \sum_{i=1}^{N} B(0,t_i)[R_{\text{fix}} - f_0(t_{i-1},t_i)](t_i - t_{i-1})L$$

where $f_0(t_{i-1},t_i)$ is the forward term rate for lending from t_{i-1} to t_i, defined by

$$f_0(t_{i-1},t_i)(t_i - t_{i-1}) = \frac{B(0,t_{i-1})}{B(0,t_i)} - 1.$$

(ii) The other approach (implicit but not explicit in the Section 10 notes) views the swap as a long position in a coupon bond paying R_{fix} plus a short position in a floating rate bond. This approach gives the formula

$$\text{swap value} = \sum_{i=1}^{N} B(0,t_i)R_{\text{fix}}(t_i - t_{i-1})L - (1 - B(0,t_N))L.$$

Show that these two approaches are consistent, i.e. the swap values given in (i) and (ii) above are equal.

3) [Hull, Chapter 22, problem 28, slightly modified.] Calculate the price of a cap on the three-month LIBOR rate in nine months' time when the principal amount is $1000. Use Black's model and the following information:

- The nine-month Eurodollar futures price is 92 (ignore the difference between forwards and futures).
- The interest rate volatility implied by a nine-month Eurodollar option is 15 percent per annum.
- The current 12-month interest rate with continuous compounding is 7.5 percent per annum.
- The cap rate is 8 percent per annum.

[See the Section 11 notes for help interpreting this jargon.]

4) [Hull, Chapter 22, problem 29] Suppose the LIBOR yield curve is flat at 8% with annual compounding. Consider a swaption that gives its holder the right to receive 7.6% in a five-year swap starting in four years. Payments are made annually. The volatility for the swap rate is 25% per annum and the principal is $1 million. Use Black's model to price the swaption.

5) [Hull, Chapter 26, problem 16, slightly modified]. Suppose the risk-free yield curve is flat at 6% with annual compounding. One-year, two-year, and three-year bonds yield 7.2%, 7.4%, and 7.6% with annual compounding. All pay 6% coupons. Assume that in case of default the recovery is 40% of principal, with no payment of accrued interest. Find the risk-neutral probability of default during each year.

6) [Hull, Chapter 27, problem 20, slightly modified.] Suppose the risk-free yield curve is flat at 6% per annum with continuous compounding, and defaults can occur at times 1 year, 2 years, 3 years, and 4 years in a four-year plain vanilla credit default swap with semiannual payments. Suppose the recovery rate is 20% and the probabilities of default at times 1 yr, 2yrs, 3yrs, and 4yrs are .01, .015, .02, and .025 respectively. The reference obligation is a bond paying a coupon semiannually of 8% per year. Assume any default takes place immediately before a coupon date, and the recovery does not include any accrued interest. What is the credit default swap spread?