Erratum concerning the article

Asset price bubbles from heterogeneous beliefs about mean reversion rates,

by X. Chen and R.V. Kohn, Finance and Stochastics (2011) 15:221-241

Theorem 5.1 of [1] draws correct conclusions, however the proof is incomplete. Indeed, the final paragraph appeals to a "basic comparison theorem for viscosity super and subsolutions, see e.g. Theorem 5.1 of [2]." Alas, the cited result from [2] concerns equations of the form $u(x) + F(Du, D^2u) - f(x) = 0$, whereas the equation under consideration in [1] does not have this form.[1]

The purpose of that final paragraph was to conclude that $\Phi(D) \leq P_*(D)$, where P_* is the minimal equilibrium price and Φ is the unique C^2 solution of

$$-\max\{\kappa_1(\theta - D), \kappa_2(\theta - D)\}\Phi' - \frac{1}{2}\sigma^2\Phi'' + \lambda\Phi - D = 0 \tag{1}$$

with linear growth at infinity. Actually, appeal to a general comparison result is unnecessary. The desired conclusion follows easily from the fact that $P_*(D)$ is a viscosity supersolution, using the asymptotic properties of $P_*(D)$ and $\Phi(D)$ as $|D| \to \infty$. Thus Theorem 5.1 of [1] can be replaced with the following:

Theorem 5.1 *The equilibrium price $\Phi(D)$ identified in Section 4 and the minimal equilibrium price $P_*(D)$ discussed in Section 3 have the following properties:*

(i) $P_*(D) \leq \Phi(D)$, *and* $\Phi(D) - P_*(D) \to 0$ *as* $|D| \to \infty$;

(ii) $P_*(D)$ *is a lower semicontinuous function; and*

(iii) $P_*(D)$ *is a viscosity supersolution of* (1).

Furthermore, assertions $(i) - (iii)$ *imply*

$$\Phi(D) \leq P_*(D), \tag{2}$$

so $\Phi = P_*$. *Thus, the unique classical solution of the differential equation with linear growth at infinity is in fact the minimal equilibrium price.*

Proof. The assertion $P_*(D) \leq \Phi(D)$ is obvious, since Φ is an equilibrium price and P_* is the minimal equilibrium price. We also know $P_*(D) \geq I(D)$ where I is the intrinsic value (characterized by (2.3) of [1]), since the definition of an equilibrium price (Definition 2.1 of [1]) includes this inequality. Theorem 4.1(b) of [1] shows that $\Phi(D) - I(D) \to 0$ as $|D| \to \infty$. This gives (i), since $\Phi(D) - P_*(D) \leq \Phi(D) - I(D)$.

Assertions (ii) and (iii) are stated and proved in Theorem 5.1 of [1].

For the final conclusion (2), consider the variational problem

$$\inf_{D \in \mathbb{R}} \{P_*(D) - \Phi(D)\}.$$

[1] We thank Yongchao Zhang for pointing this out.

If a minimizing sequence tends to $\pm\infty$ then the minimum value is 0 by (i), and (2) is true. If on the other hand a minimizing sequence stays bounded, then the minimum is achieved at some D_1, by (ii). Since P_* is a viscosity supersolution we have

$$-\max\{\kappa_1(\theta - D_1), \kappa_2(\theta - D_1)\}\Phi'(D_1) - \frac{1}{2}\sigma^2\Phi''(D_1) + \lambda P_*(D_1) - D_1 \geq 0.$$

It follows since Φ solves (1) that

$$-\lambda\Phi(D_1) + \lambda P_*(D_1) \geq 0.$$

Since the discount rate λ is positive, we conclude that $P_*(D_1) - \Phi(D_1) \geq 0$. Thus $P_*(D) - \Phi(D) \geq P_*(D_1) - \Phi(D_1) \geq 0$, completing the proof of (2). □

References

[1] Chen, X., Kohn, R.V.: Asset price bubbles from heterogeneous beliefs about mean reversion rates. Finance Stoch. **15**, 221-241 (2011)

[2] Crandall, M.G., Ishii, H., Lions P.L.: User's guide to viscosity solutions of second order partial differential equations. Bull. Am. Math. Soc. **27**, 1-67 (1992)