Some types of questions that might be on the final exam. This list is exemplary, not comprehensive.

- Section 1 material:
  - Consider the following optimal control problem.... Identify an appropriate value function, and specify the Hamilton-Jacobi equation it solves. Assuming an optimal control exists, how should it be related to the value function?
  - Consider the following optimal control problem ... with value function u, and the associated Hamilton-Jacobi equation ... Show that if v is differentiable and it solves the Hamilton-Jacobi equation ... then $u \leq v$.
- Section 2 material:
  - Recall the definition of a viscosity solution of $u_t + |u_x| = 1$. (Repetition of the two-part definition here.) Consider the function $v(x,t) =$ explicit formula, which solves the equation in the classical sense wherever it is differentiable. Is it a viscosity solution? Why or why not?
  - Explain why a smooth solution of a Hamilton-Jacobi equation is automatically also a viscosity solution.
- Section 3 material:
  - Consider the following stochastic optimal control problem... Identify an appropriate value function, and specify the Hamilton-Jacobi-Bellman equation it solves. What can you say about an optimal control policy?
  - Consider the Merton portfolio problem with utility c^γ (statement of problem here). Show (without finding or using the explicit solution) that the value function satisfies $u(ax,t) = a^\gamma u(x,t)$ for any a.
  - Explain the following statement: "Considering the viscosity solution of a Hamilton-Jacobi equation with a convex Hamiltonian corresponds to perturbing the associated optimal control problem by introducing a little bit of randomness." You may focus a specific problem or class of problems rather than the most general case.
  - Consider the following stochastic control problem ... with value function u, and the associated Hamilton-Jacobi-Bellman equation ... Show that if v is differentiable and it solves the equation ... then $u \leq v$.
- Section 4 material:
  - Show, using the definition of the stochastic integral, that $E\left[\left(\int_0^T f\,dw\right)^2\right] = \int_0^T E\left[f^2\right] dt$.
- Section 5 and Section 5 addendum material:
  - Applications of the Ito calculus similar to the problems in HW 4.
  - What PDE describes the probability that ....? (e.g. like HW 4 problem 7).
  - Consider the pde ..... For what stochastic process is it the forward Kolmogorov equation? What is the probabilistic interpretation of its solution?
  - Consider the stochastic differential equation.... Specify its forward and backward Kolmogorov equations. Show that if u solves the backward equation then $E[u(y(s),s)]$ is independent of s (you may use the fact that the evolving probability density satisfies the forward equation).

- Section 6 material:
  - Consider the following problem for the linear heat equation ... Is there a solution? Is it unique? Justify your answer.
  - Suppose $u_t - u_{xx} + a(x,t)u = 0$ for $0 < x < 1$, with $u(0,t) = u(1,t) = 0$ for all $t > 0$. Suppose in addition $u > 0$ at $t = 0$. Show that $u > 0$ for all t.