Partial Differential Equations for Finance
G63.2706, Spring 2000
Tuesdays 5:10-7pm
810 Main

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Teaching Assistant: Juyong Lim. Office: 809 Phone: 998-3207. Email: limju@cims.nyu.edu. Office hours: Mon 6-7 and Wed 12-1 (by default in her office, but using 813 when a classroom setting is useful).

Special Dates: First lecture Jan. 18. No lecture March 14 (spring break) and April 4 (out of town). Last lecture April 25. Final exam: May 9.

Content: An introduction to those aspects of partial differential equations most relevant to finance. Deterministic optimal control: dynamic programming, Hamilton-Jacobi equations, verification theorems, applications. Stochastic optimal control: the Hamilton-Jacobi-Bellman equation and various applications in discrete and continuous time. Relation between diffusion processes and PDEs: stochastic differential equations, forward and backward Kolmogrov equations, Feynman-Kac formula, stopping times and free boundary problems. Parabolic equations: fundamental solution, regularizing property, maximum principle, boundary conditions, barrier options. The inverse problem of finding volatility from option prices.

Prerequisites: Calculus and discrete probability. No specific knowledge of differential equations or stochastic processes will be assumed. However students will need considerable scientific maturity, at a level typically obtained through a firm undergraduate background in economics or the physical sciences. Students without any prior exposure to stochastic differential equations will have to do some reading: chapters 9, 10, and 11 of Neftci's book are more than enough. (Mainly I'll assume some familiarity with Ito's lemma.)

Course requirements: There will be several homework sets, one every couple of weeks, probably 5 or 6 in all. Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually. There will be one in-class final exam.

Lecture notes: Lecture notes and homework sets will be handed out, and also posted on my website as they become available. (Go to the CIMS home page www.cims.nyu.edu and select faculty to find me.)

Books: There is no textbook for this course. The following is a list of books that have been ordered by the NYU bookstore and placed on reserve in the CIMS library.

- B.K. Oksendal, *Stochastic differential equations: an introduction with applications* (5th edition, paper) Springer-Verlag, 1998. Much deeper and more sophisticated than this course, but closely related. A great book, well worth reading if you have the background. My treatment of the backward and forward Kolmogorov equations, and of optimal stopping, is mainly from here.
- W. Fleming and R. Rishel, *Deterministic and stochastic optimal control*, Springer-Verlag, 1975 (reprinted with corrections recently). Again, much deeper and more sophisticated than this course but closely related. Harder to read than Oksendal, and a little out of date.

- L.C. Evans, *Partial Differential Equations*, American Math Society, 1998. This is the best modern first-year-PhD-level PDE text I know. It includes an elementary discussion of deterministic optimal control and Hamilton-Jacobi equations – going far beyond what we'll do in class (for example, this is a fine place to read about viscosity solutions). Also good for basic material on parabolic equations. Strongly recommended; however we'll be covering just a small fraction of this book.

- S. Neftci, *An introduction to the mathematics of financial derivatives*, Academic Press, 1996. This book is perhaps more basic than the course. It doesn't do much PDE. But provides a good, not-too-technical introduction to stochastic integrals, the Ito calculus, and related topics, including a good discussion of why diffusions play such a central role in financial models

- P. Wilmott, J. Dewynne, and S. Howison, *The mathematics of financial derivatives: a student introduction*, Cambridge Univ Press, 1995. This book avoids almost all discussion of diffusion processes associated with option pricing, focusing instead as much as possible on the associated PDE's. Relatively easy to read; goes much further than this class on numerical approximation schemes, American options, etc. [This is the item ordered in the bookstore. The item on reserve is an earlier, somewhat more comprehensive book by the same authors.]

Which, if any, should you buy? That depends on your background and interests. Look carefully at Oksendal and Fleming/Rishel before buying them: these are very good books, but you may not have the background to read them (particularly the latter).

The following books have been placed on reserve in the CIMS library but have *not* been ordered by the bookstore.

- I. Capuzzo Dolcetta and P.-L. Lions, editors, *Viscosity solutions and applications*, Lecture Notes in Mathematics No. 1660, Springer-Verlag, 1997. A sophisticated book, far beyond the level of this course. Two chapters are of interest: one by Bardi ("Some applications of viscosity solutions to optimal control and differential games") and one by Soner ("Controlled Markov processes, viscosity solutions, and applications to mathematical finance").

- M. Baxter and A. Rennie, *Financial calculus: an introduction to derivative pricing*, Cambridge University Press, 1996. An extremely readable, relatively nontechnical treatment of derivative pricing from the viewpoint of martingales and risk-neutral measures. Includes a good, if brief, summary of stochastic calculus. Valuable background for students taking this course who have not yet taken Math Finance I and II.

- F. John, *Partial differential equations*, 4th edition, Springer-Verlag. This was the best basic-level introduction before Evans' book appeared. Good treatment of parabolic PDE, including basic material about numerical discretization. But no control theory or dynamic programming.

- Dimitri Bertsekas, *Dynamic programming: deterministic and stochastic models*, Prentice-Hall, 1987. No PDE here either, and too comprehensive, but the early chapters give enlightening examples of discrete-time dynamic programming.

- Jack Macki and Aaron Strauss, *Introduction to optimal control theory*, Springer-Verlag. An undergraduate-level text on deterministic optimal control (focusing mainly on applications in the physical sciences).

I will from time to time place xeroxed articles or sections from books on reserve. They'll be in the "Green box" with my name on it, which you can request from the CIMS library staff.