Partial Differential Equations for Finance
G63.2706, Spring 2011
Mondays 5:10-7pm
WWH 517

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Special Dates: First lecture 1/24. No lecture 2/21 (Presidents' Day) and 3/14 (Spring Break). The final exam will be 5/9 (the last class day, rather than exam period). Peter Carr will give a guest lecture (tentatively 3/7).

Content: An introduction to those aspects of partial differential equations and optimal control most relevant to finance. PDE's naturally associated to diffusion processes: the forward and backward Kolmogorov equations and their applications. Linear parabolic equations: fundamental solution, boundary value problems, maximum principle. Dynamic programming and optimal control: Hamilton-Jacobi-Bellman equation, verification arguments, optimal stopping. Applications to finance will be distributed throughout the course, including: barrier options; options on an underlying that can jump; portfolio optimization; American options; other examples of optimal decision-making.

Prerequisites: Working knowledge of stochastic calculus, and basic familiarity with the Black-Scholes approach to option pricing.

Course requirements: There will be several homework sets, one every couple of weeks, probably 6 in all. Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually. There will be one in-class final (on the last official class day, not during exam period).

Lecture notes: Lecture notes and homework sets will be handed out, and also posted on my web-site as they become available. HW solutions will be posted on an associated BlackBoard site.

Semester plan: Lectures 1-2: the backward and forward Kolmogorov equations and their applications. Lectures 3-4: the linear heat equation – properties and solution formulas on the real line, a half-line, and an interval. Lectures 5-6: jump processes, and a guest lecture by Peter Carr on barrier options. Lectures 7-8: deterministic optimal control and dynamic programming. Lectures 9-11: stochastic optimal control including portfolio optimization, optimal stopping, and American options. Lectures 12: the martingale approach to portfolio optimization. Lecture 13: asset price bubbles from heterogeneous expectations.

Text: There is no text for this class – the right book doesn't exist. However all lectures will be supported by lecture notes or readings. See a separate handout for a list of books that correlate with various parts of the class.