Problem set 10 - Due 05/14/2012
Functional analysis - spring 2012

1) If M is a closed subspace of a seperable normed vector space X, show that X/M is also seperable.

2) Show that the range of a compact operator is seperable.

3) For X a B-space, show that x_k converges strongly to x and x'_k converges weak* to x' in X' implies that $x'_k(x_k)$ converges to $x'(x)$.

a/ Prove that the same holds if x_k converges weakly to x and x'_k converges strongly to x'.

b/ Is it still true if x_k converges weakly to x and x'_k converges weak* to x' in X'.

4) $X = l_\infty$ and let δ_n be given in X^* by

$$\delta_n(\{c_k\}_{k=1}^\infty) = c_n.$$

Prove that $\{\delta_n\}$ has no weak* convergent subsequence.

[extra pb: For those who know the notion of nets and subnets, you can prove that $\{\delta_n\}$ has a weak* convergent subnet.]

5) If E is seperable, then the unit ball of E' is metrizable for the weak* topology.

6) Prove that $L^2(-\infty, \infty)$ is serperable.

7) Show that A is normal if and only if $AA^* = A^*A$, namely A and A^* commute.

8) Let A be a selfadjoint operator on $B(H)$ and m and M are givem by

$$m = \inf_{||u||=1} (Au, u), \quad M = \sup_{||u||=1} (Au, u).$$

Show that both m and M are in $\sigma(A)$.

9) Let A be a normal operator. Prove that $A = B + iC$ where B and C are commuting selfadjoint operators.