Problem set 3 - Due 03/19/2012
Functional analysis - spring 2012

1) Show that if X is infinite dimensional and K is one-to-one operator of $K(X)$ then $I - K$ cannot be in $K(X)$.

2) Let $A \in B(X, Y)$ and $K \in K(X, Y)$ where X and Y are B-spaces. If $R(A) \subset R(K)$, show that $A \in K(X, Y)$.

3) Let X and Y be B-spaces. Let A be an operator in $B(X, Y)$ such that $R(A)$ is closed and infinite dimensional. Show that A cannot be compact.

4) Show that a linear functional on a B-space X is bounded if and only if $N(T)$ is closed.

5) Show that every linear space can be normed.

6) If M is a closed subspce of a B-space X, then X/M is a B-space

Extra problem:

7) Let X and Y be B-spaces. Let A be a closed linear operator from X to Y. Prove that $R(A)$ is closed in Y if and only if there exists a C such that

$$d(x, N(A)) \leq C||Ax||, \qquad x \in D(A).$$