Problem set 4 - Due 03/26/2012
Functional analysis - spring 2012

1) Show that a linear functional A on a B-space is bounded if and only if $N(A)$ is closed.

2) Let $T \in B(X,Y)$. Then T is compact if and only if $[T] \in B(X/N(T),Y)$ is compact.

3) Let $X = l^2$. Define $T \in B(X)$ by $T(x_1, x_2, ...) = (0, x_1/1, x_2/2, ..., x_n/n, ...)$. Prove that T is compact.
Extra question: Prove that it has no eigenvalues.

4) Let $k(x,y)$ be a continuous function on $[0,1]^2$. Define Tf by

$$Tf(x) = \int_0^1 k(x,y)f(y)dy$$

for any integrable function on $[0,1]$. Show that for any $1 \leq p \leq \infty$, T is compact from $L^p(0,1)$ to itself.

5) Let N be a finite dimensional subspace of a normed vector space X.
Prove that there exists a closed subspace X_0 such that $X = X_0 \oplus N$.

6) Let R be a closed subspace of a normed vector space X such that R^o is of finite dimension n.
Prove that there is an n diminsional subspace N of X such that $X = R \oplus N$.