Problem set 5 - Due 04/02/2012
Functional analysis - spring 2012

1) Show that if there is a nonnegative k such that A^k is compact then $I - A$ is Fredholm.

2) X and Y are B-space. Show that if $A \in B(X,Y)$ and $A' \in K(Y', X')$, then $A \in K(X,Y)$.

3) Suppose X is a B-space, $\dim X = \infty$ and let $A \in K(X)$. Show that $0 \in \sigma(A)$ and that if $R(A)$ is infinite dimensional, it is never closed.

4) Let $X = l^2$ and define $T : X \to X$ by

$$T(x_1, x_2, ...,) = (0, x_1/1, x_2/2, ..., x_n/n, ...).$$

Show that T is compact. What is the spectrum of T ?

Same question for

$$S(x_1, x_2, ...,) = (x_1/1, x_2/2, ..., x_n/n, ...).$$

5) $T \in B(X)$ and satisfies $T^2 = I$. Show that $\sigma(T) \in \{1, -1\}$.

Show that if $T \neq \pm I$, then $\sigma(T) = \{1, -1\}$ and moreover we have the direct sum decomposition $X = N(T - I) \oplus N(T + I)$.

6) Let K be of rank 1, namely $K(x) = x_1'(x)x_1$ for some $x_1 \in X$ and $x_1' \in X'$. What is $\sigma(K)$?