Problem set 7 - Due 04/16/2012
Functional analysis - spring 2012

1) If M is a subspace of a B-space X, we recall that M has finite codimension if $\text{codim}M = \dim(X/M) < \infty$. Show by an example that in this case, M is not necessary closed.

2) X and Y are B-spaces and $A \in B(X,Y)$. Assume that $\text{codim}(T) = \text{codim}(Y/R(A)) < \infty$. Show that $R(A)$ is closed.

3) Let X be a B-space and $A \in B(X)$. We recall that λ is in the residual spectrum if λ is not an eigenvalue and $R(\lambda - A)$ is not dense. Show that :
a) If λ is in the residual spectrum of A, then λ is in the point spectrum of A'
b) If λ is in the point spectrum of A, then λ is either in the point or residual spectrum of A'.

4) What is the spectrum of the shift operator on l^1:

$$A(x_1, x_2, ...,) = (x_2, x_3, ...).$$

What is A' ? [This can be a good example to understand pb 3)]

5) Give an exampe of a bounded operator such that such that the range is not close. Prove that if A is bounded, everywhere defined and an isometry, then $R(A)$ is closed.