Problem set 8 - Due 04/23/2012
Functional analysis - spring 2012

1) Suppose λ is an isolated point of $\sigma(A)$, $A \in B(X)$ and $f(z)$ is analytic in a neighborhood of λ. If $f(A) = 0$ show that $f(z)$ has a zero at λ.

2) Suppose $A \in B(X)$ and $\sigma(A)$ is contained in the half plan $\mathrm{Re}(z) > \delta > 0$. Let Γ be a simple closed curve in $\mathrm{Re}(z) > \delta$ containing $\sigma(A)$ in its interior. Consider

$$T = \frac{1}{2\pi i} \int_\Gamma z^{1/2}(z - A)^{-1} dz.$$

Show that T is well defined and that $T^2 = A$. What is $\sigma(T)$.

3) Suppose $A, B \in B(X)$ and $0 \in \rho(A)$, $\|A - B\| < 1/\|A^{-1}\|$. Show that $0 \in \rho(B)$ and

$$\|B^{-1}\| \leq \frac{\|A^{-1}\|}{1 - \|A^{-1}\|.\|A - B\|}$$

4) Let A be linear with $D(A)$ dense in X. If $B \in B(Y, Z)$, show that $(BA)' = A'B'$.

5) Let A be an unbounded operator from X to Y with domain $D(A)$ which is injective. Consider the following additional properties

a) A is a closed operator

b) $R(A)$ is dense

c) $R(A)$ is closed

d) there exists C such that for all $x \in D(A)$, $\|Ax\| \geq C\|x\|$

Prove that a), b) and c) imply d)

Prove that b) , c) and d) imply a)

Prove that a) and d) imply c) .

The grader can be contacted at : Tarek Elgindi : elgindi@access1.cims.nyu.edu