Complex Analysis Fall 08, practice test
Complex analysis

Justify your answers.

1. What are all the the conformal mapping from the disc $D(0,1)$ to the disc $D(0,r)$, $r > 1$.

2. Compute the integral
$$\int_0^\infty \frac{1}{x^3 + x + 1} dx.$$

3. Prove that two circular annuli are conformally equivalent (there exists a conformal map from one into the other) if and only if the ratios of their radii are equal.

4. Suppose f is C^2 and subharmonic on the unit disc. Define
$$F(r) = \int_0^{2\pi} f(re^{i\theta}) d\theta.$$

   Prove that F is nondecreasing function of r.

5. If f is analytic in the upper half plane $\{Im(z) > 0\}$ and bounded by M. Find a bound on sup of $f^{(n)}$ in the half plane $\{Im(z) > r\}$ for $r > 0$.