Score:	Name:

HW10 - Due 04/16/2008
ODE - spring 2008

1) For $\epsilon > 0$, approximate the solution of

$$x'' + x - \epsilon x^3 = 0$$

with $x(0) = 1, x'(0) = 0$ till the order ϵ^2 on a fixed time interval. (The difference is a $O(\epsilon^3)$).

2)Take

$$\begin{cases} x' = x - x^2 - \epsilon xy \\ y' = y - y^2 - \epsilon xy \end{cases} \tag{1}$$

1/Starting with positive initial values $x(0)$ and $y(0)$, compute the expansion of the solution till the order ϵ.
2/ On which time interval is this approximation valid.

3) Determine the stability of of $(0, 0)$ for

$$x'' + x^n = 0 \tag{2}$$

where $n \in \mathbb{N}$.

4) Determine the stability of $(0, 0)$ for

$$\begin{cases} x' = 2xy + x^3 \\ y' = x^2 - y^5. \end{cases} \tag{3}$$

Rk : For references about the perturbation theory
E. A. Coddington and N. Levinson, Theory of ordinary differential equations, McGraw-Hill, 1955
F. Verhulst, Nonlinear Differential equations and Dynamical systems.