Score:	Name:

HW11 - Due 04/23/2008
ODE - spring 2008

This HW will count as 1/3 of the final grade.

1) Solve $x'' + \frac{1}{4t^2}x = 0$ with $x(1) = 1$ and $x'(1) = 0$.

2) Consider the system

$$x' = Ax + f(t,x) + \mu g(t)$$

where A is a constant matrix, f and g are continuous and T-periodic in t, f_x exists and is continuous. Assume that $y' = Ay$ has no nontrivial solution of period T and that $f_x(t,0) = 0$ and $f(t,0) = 0$. Prove that for small μ, there exists a unique solution $\phi(t,\mu)$ of period T which is continuous in (t,μ) for small μ.

3) Prove that if $\int_0^\infty |A(t)|dt < \infty$ where A is n by n matrix, then any nontrivial solution of $x' = A(t)x$ where $x \in \mathbb{R}^n$ has a limit different from zero when t goes to infinity. Prove that this defines a bijection between the initial data at $t = 0$ and this limit.

4) Consider $y' = Ay$, $y \in \mathbb{R}^n$ and A is n by n matrix, and assume that $|e^{tA}| \leq M_0$ for $t \geq 0$. let $f(t,x)$ be such that $|f(t,x)| \leq g(t)|x|$ for $t \geq 0$ and $\int_0^\infty g(t) < \infty$.

a/ Show that there exists a constant M such that any solution ϕ of $x' = Ax + f(t,x)$ satisfies $|\phi(t)| \leq M|\phi(0)|$

b/ If p is the number of eigenvalues of A with zero real part. Prove that there exists a p dimensional space P in $\mathbb{R}^n$ such that for each solution ϕ of $x' = Ax + f(t,x)$, there exists a unique $q \in P$ such that $\phi(t) - e^{tA}q \to 0$ when t goes to infinity.

[Hint: Recall that e^{tA} can be writen as $e^{tA} = U_1(t) + U_2(t)$ where $|U_2(t)| \leq Ke^{-\sigma t}$ for $0 \leq t < \infty$ and $\sigma > 0$ and $|U_1(t)| \leq K$]

PS: Please check for up dated versions if there are any corrections.