

Score:

Name:

HW12 - Due 04/30/2008
ODE - spring 2008

This HW will count as 1/3 of the final grade.

1) Study the stability of the critical points of $x' = 1 - 2\mu x + x^2$ where μ is a parameter.

2) Consider $x'' + \sin(x) = 0$ with $x(0) = a$ and $x'(0) = 0$ and $0 < a < \pi$.

a/ Prove that the solution is periodic in time with period $T(a)$

b/ Find an expansion of $T(a)$ when a goes to 0 (just compute the first two terms)

c/ What is the behaviour of $T(a)$ when a goes to π .

d/ What happens for other values of $a \in \mathbb{R}$.

3) Consider the following system with one prey and two predators

$$\begin{cases} x' = ax - xy - xz \\ y' = -by + xy \\ z' = -cz + xz \end{cases} \quad (1)$$

with $a, b, c > 0$. Take an initial data $x(0), y(0), z(0) \geq 0$.

a/ Prove that the system has a global solution and that for $t \geq 0$, we have $x(t), y(t), z(t) \geq 0$.

b/ What are the equilibrium solutions. Characterise them

c/ Are there periodic solutions with $y(t)z(t) > 0$?

d/ Is it possible that starting from $y(0), z(0) > 0$, one of the predators becomes extinct in finite time ? or in infinite time ?

PS: Please check for up dated versions if there are any corrections.