

Score:

Name:

**HW1 - Due 02/20/2008**  
**ODE - spring 2008**

1) For the following two dimensional system in  $\mathbb{R}^2$

$$\begin{cases} x' = y(1 + x - y^2) \\ y' = x(1 + y - x^2) \end{cases} \quad (1)$$

determine the critical points and characterise the linearised flow in a neighbourhood of the these points.

2) What is the attraction property of the solution  $(0, 0)$  of

$$\begin{cases} x' = x^3 + y \\ y' = (x^2 + y^2 - 2)y \end{cases} \quad (2)$$

3) Consider the equation  $x'' - \lambda x' - (\lambda - 1)(\lambda - 2)x = 0$  with  $\lambda$  a parameter. Find the critical points and characterise them. Sketch the flow in the phase-plane for  $\lambda = -1, 0, 1$  and  $3$ .

4) Solve the following equation

1.  $x' = \frac{x^2 - t^2}{x^2 + t^2}$

you can use polar coordinates.