Score:		Name:

HW7 - Due 03/26/2008
ODE - spring 2008

1)If $\phi_1, \phi_2, ..., \phi_n$ is a fundamental set for the homogeneous equation

$$L(x) = x^{(n)} + a_1 x^{(n-1)} + ... + a_n x = 0 \tag{1}$$

where $a_1, a_2, ..., a_n \in C(I)$ are continuous functions of t, then find the solution of $L\psi = b(t)$ where $b \in C(I)$ and $\psi(\tau) = \psi'(\tau) = \psi^{(n-1)}(\tau) = 0$, $\tau \in I$.

2) Determine the stability property of the solution $(sin(t), cos(t))$ of the system

$$\begin{cases} x' = +y(x^2+y^2)^{-1/2} \\ y' = -x(x^2+y^2)^{-1/2} \end{cases} \tag{2}$$

3) Prove that the equation

$$x'' + 2\mu x' + \omega_0^2 x = sin(\omega t) \tag{3}$$

where $0 < \mu < \omega_0$ and $\omega > 0$ has a periodic solution and study its stability.

4) Consider the second order equation

$$x'' + x^2 + bx + c = 0 \tag{4}$$

where b and c are two parameters.

a/ Determine the critical points and their stability.

b/ Give a condition for periodic solutions to exist. Are these solution Lyapunov-stable or orbitally stable or asymptotically stable ?