

Score:

Name:

**HW8 - Due 04/02/2008**  
**ODE - spring 2008**

1) What are the stability properties of the  $(0, 0)$  solution of

$$\begin{cases} x' = \alpha x + y^n \\ y' = \alpha y - x^n \end{cases} \quad (1)$$

depending on the parameters  $\alpha \in \mathbb{R}$  and  $n \in \mathbb{N}$ .

2) Show by an example that if  $f$  is  $C^1$  and  $f(0) = 0$ , it is possible that  $\lim_{t \rightarrow \infty} x(t) = 0$  when  $t$  goes to  $+\infty$  for all solutions to  $x' = f(x)$  without the eigenvalues of  $Df(0)$  having negative real parts.

3) Consider

$$\begin{cases} x' = 1 + y - x^2 - y^2 \\ y' = 1 - x - x^2 - y^2 \end{cases} \quad (2)$$

a/ Determine the critical points and characterise them

b/ Show that the system has a periodic solution

c/ Linearize the system near this periodic solution and determine the characteristic exponents.