HW8 - Due 04/09/2008
ODE - spring 2008

1) Consider the system

$$\begin{cases} x' = -y + f(x,y) \\ y' = \sin(x) \end{cases} \tag{1}$$

where the function f is smooth.
Give some sufficient condition on f so that $(0,0)$ is a stable equilibrium

2) Consider

$$x'' + \phi(t)x = 0 \tag{2}$$

with $\phi(t) \in C^1(\mathbb{R})$ monotonic and such that $lim_{t\to\infty}\phi(t) = c > 0$.
Prove that $(0,0)$ is stable.

3) Consider again the system :

$$\begin{cases} x' = 1 + y - x^2 - y^2 \\ y' = 1 - x - x^2 - y^2 \end{cases} \tag{3}$$

Only answer question d/ :
a/ Determine the critial points and characterise them
b/ Show that the system has a periodic solution
c/ Linearize the system near this periodic solution and determine the characteristic exponents.
d/ determine the stability of the periodic solution constructed in c

4) The equation

$$x'' - (1 - x^2 - (x')^2)x' + x = 0 \tag{4}$$

has a critical point and a limit cycle
Determine them and characterise them