Score:	Name:

HW1 - Due 02/24/2015
ODE - spring 2015

1) For the following two dimensional system in $\mathbb{R}^2$

$$\begin{cases} x' = y(1 + x - y^2) \\ y' = x(1 + y - x^2) \end{cases} \tag{1}$$

determine the critical points and write down the linearised flow in a neighbourhood of the these points. Solve the linearized problem.

2) Find all solutions to the equation $x' = \sqrt{|x|}$ such that $x(t = 0) = 0$.
What is the solution and the time of existence of the equation $x' = \sqrt{|x|}$ with $x(t = 0) = x_0 > 0$.

3) Consider the equation $x'' - \lambda x' - (\lambda - 1)(\lambda - 2)x = 0$ with λ a parameter. Find the critical points and characterise them.
(optional) Sketch the flow in the phase-plane for $\lambda = -1, 0, 1$ and 3.

4) Solve the following equation

1. $x' = \frac{x^2 - t^2}{x^2 + t^2}$

you can use polar coordinates.