# 1) OUTLINE

(a) NP-Hard to determine if a graph has an embedding with Resolution $2\pi/d$.

(b) Resolution of outerplanar graphs.

(c) upper bound on the resolution of a Random graph.

# 2) Review

(a) Resolution: Minimum angle between edges incident on the same vertex in a straight line embedding of the Graph.

# 2) NP- HARDNESS

(a) What is : NP-Complete Problem?
: NP- Hard Problem?

(b) How to Prove a Problem is NP-Hard/complete?

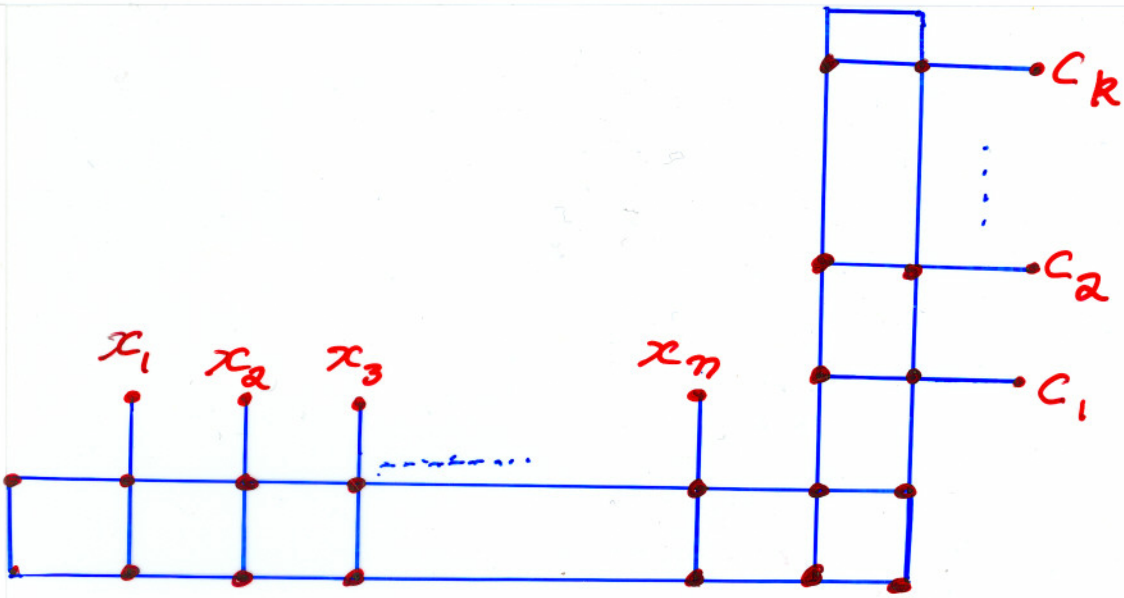
(c) Example of NP-Complete Problem: SAT, 3 SAT

**Theorem**

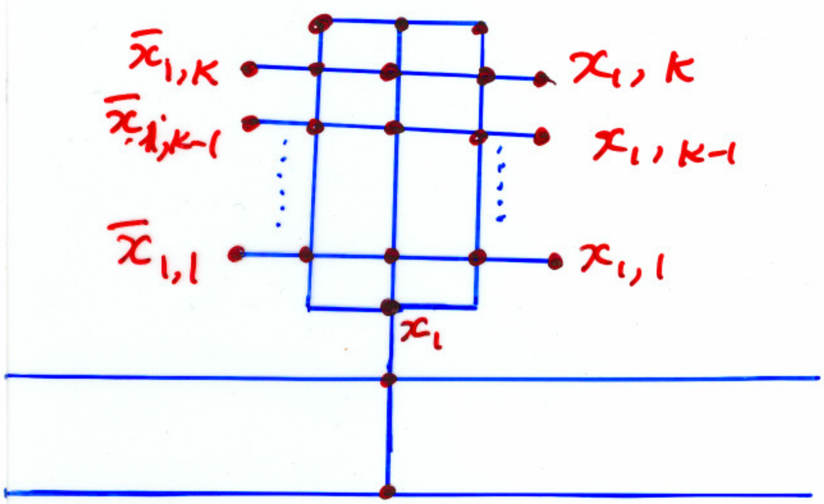
Given a graph G, of max degree 4, the decision Problem of whether or not G can be embedded in the plane with resolution $\pi/2$ is NP- HARD.

**Proof**:

Reduction from 3-SAT.



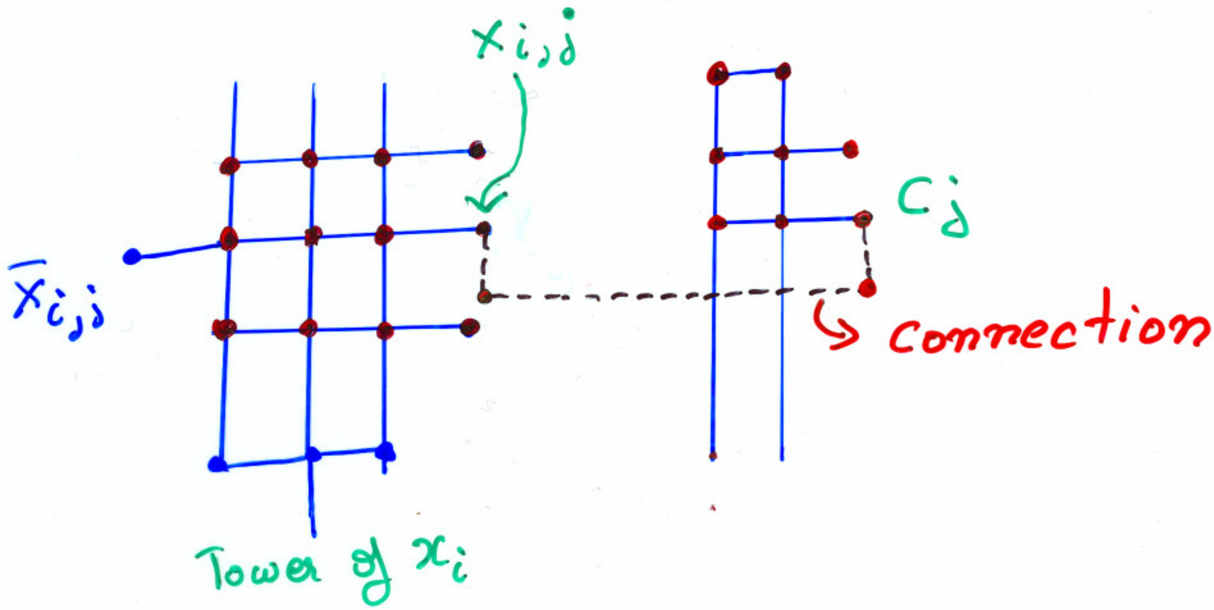

The skeleton.




The Tower

(1) Place a tower on each variable in the skeleton.

(2)




clause $c_j$ contains $x_i$

**Note:**

(i) Each clause has at most 3 Connections.

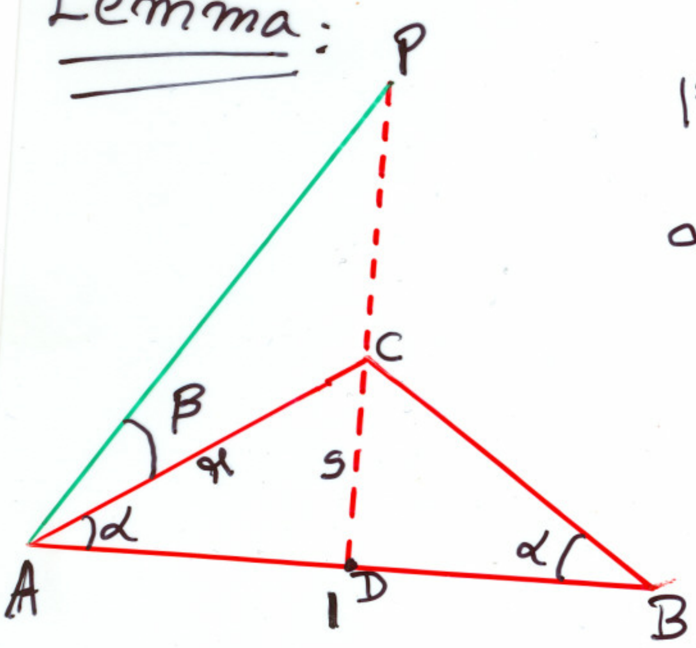
(ii) In how many ways can the Connections be embedded?

# Angular Resolution of Outerplanar Graphs

**Theorem**

Every triangulated outerplanar graph with max degree d has a planar straight line embedding where each face is an isoceles triangle with base angle $= \frac{\pi}{2(d+2)}$

**Lemma:**

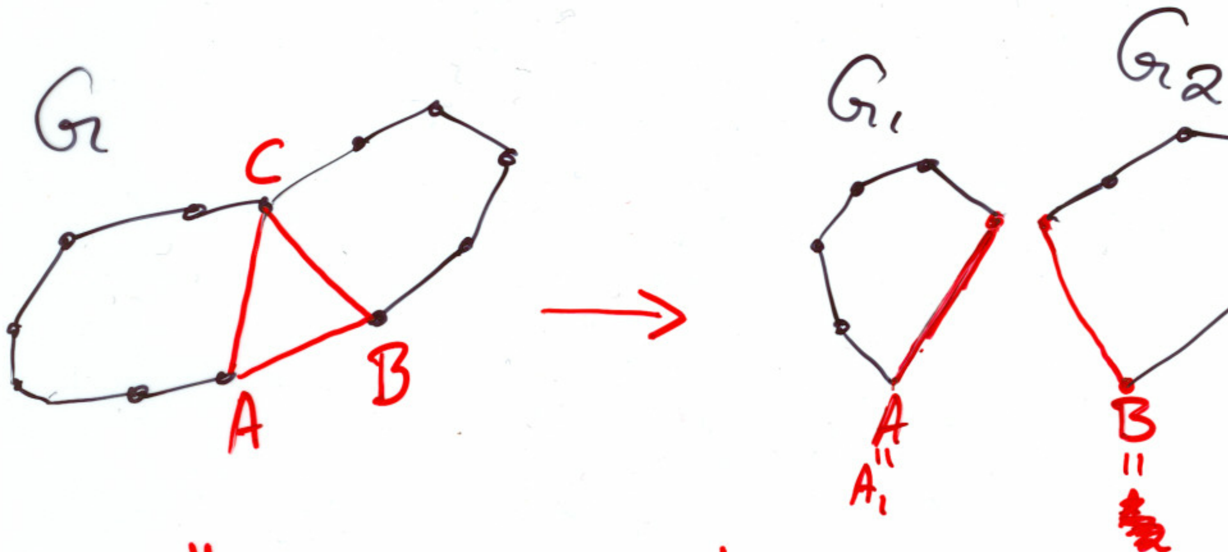



$$|PD| = l = s\sum_{i=0}^{\infty} r^i$$

$$\alpha = \frac{\pi}{2(t+2)} \quad \text{where } t \geq 2$$

Then:

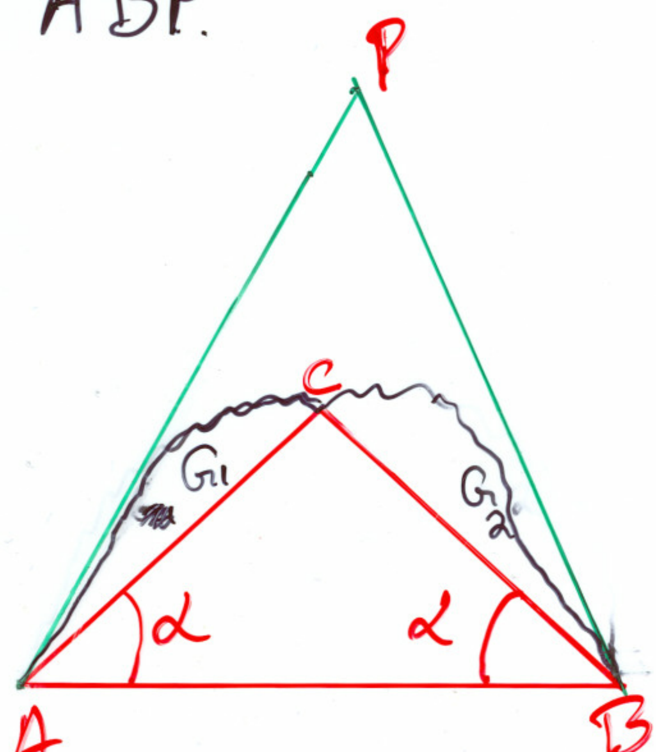
$$(t-1)\alpha + \beta < \pi/2$$




we will Prove a stronger statement:-

Additional Property:

Embedding of $G$ is contained inside triangle $ABP$.




Also: $(\text{degree}(A)-1)\alpha + \angle PAC < \pi/2$

# Resolution of a Random Graph:-

A "Handwaving" Proof!!

**Theorem:**

"Many" graphs with max degree $d$ have resolution at most $O\left(\frac{\log d}{d^2}\right)$

**Proof:**

(i) For simplicity consider directed graphs with out degree $d$.

(ii) Consider only angles formed by out going edges.

(iii) Number of $N$-node out degree $d$ graphs:

$$\binom{N-1}{d}^N$$

## Lemma

Given any set of N points in the plane, it is possible to find concentric circles with radii $r_1$ and $r_2$ such that at least $N/5$ points are inside or on the boundary of the inner circle and atleast $N/5$ are outside or on the boundary of the outer circle where $r_2 \geq r_1$