



## Anomalous Hydrodynamic Drafting of Interacting Flapping Flags

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In aggregates of objects moving through a fluid, bodies downstream of a leader generally experience reduced drag force. This conventional drafting holds for objects of fixed shape, but interactions of deformable bodies in a flow are poorly understood, as in schools of fish. In our experiments on “schooling” flapping flags, we find that it is the leader of a group who enjoys a significant drag reduction (of up to 50%), while the downstream flag suffers a drag increase. This counterintuitive inverted drag relationship is rationalized by dissecting the mutual influence of shape and flow in determining drag. Inverted drafting has never been observed with rigid bodies, apparently due to the inability to deform in response to the altered flow field of neighbors.

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Groupings of bodies moving through a fluid often exhibit coordinated behavior. Locomotion provides well-known examples including the maneuvering and clustering of racing automobiles [1] and bicyclists [2] and the queuing of lobsters during underwater migrations [3]. These phenomena are explained by conventional hydrodynamic drafting, for which rigid bodies enjoy drag reduction when situated behind a leader in a flow [4]. Though the flow field around drafting bodies is complicated, the effect is qualitatively understood by considering that the downstream body sits in the lower velocity wake of the leader. The resulting force arrangement leads to passive aggregation and offers net drag reduction for locomotors.

Does this rationalization of interactions among rigid objects extend to aggregates of flexible, shape-changing bodies? Schooling fish [5] and flocking birds [6] are striking examples in which fluid and dynamic structures conspire to support long length-scale coherent motion [7]. Understanding such phenomena is challenging because of the mutual influence of shape and flow [8–10]. The flapping of a flag is an everyday example that serves as an archetype of such problems [10]. Certainly, the changing shape of a flexible body affects the resistive force it must overcome when moving through a fluid [9]. Moreover, when deformable bodies condense into an aggregate state, fluid-mediated interactions may lead to modifications of shape and, hence, modifications in force. Here, we reveal an instance of the dramatic effect of deformability in the interaction of flapping flags in a fast flow.

To experimentally model the interaction of shape-changing bodies, we insert thin flexible filaments into a flowing soap film [11]. Each filament is fixed at its upstream end to a thin wire, a flagpole that extends out of the film, while the rest of the thread hangs free in the film. Because the filaments are sufficiently flexible and massive, they spontaneously flap under the fluid forcing, as one-

dimensional flags fluttering in a two-dimensional breeze [10,13,14]. These threads have diameter 0.03 cm, length  $L = 2.0$  cm, bending modulus  $0.34 \text{ g cm}^3/\text{s}^2$ , and mass per unit length  $4.8 \times 10^{-4} \text{ g/cm}$ ; they flap in a flow of far-field speed  $U = 200 \text{ cm/s}$ . First, consider the undulations of a lone flag, whose motion is well described by a traveling wave of increasing spatial envelope [10]. For the above material and flow properties, the motion of a flag exhibits maximum amplitude  $A_0 = 1.36 \text{ cm}$  (the total excursion of the free end) and flapping frequency  $f_0 = 35.8 \text{ Hz}$ . In addition to these kinematic quantities, we measure the time-averaged streamwise fluid force,  $D_0 = 5.2 \text{ g cm/s}^2$ , the drag on an isolated flag [15].

We then insert a second flag into the flow. Depending on the relative location of the bodies, each may be presented with a modified flow field, which in turn may alter the form of the flapping motion [10,16,17]. In these experiments, we capture the amplitude  $A$  and frequency  $f$  on each body, as well as the fluid force  $D$ . At the high Reynolds number ( $\text{Re} \sim 10^4$ ) studied here, the primary influence of an object on a flow is downstream, yielding the complex wake of the body [18]. Thus, the queuing of bodies in the direction of flow is the simplest arrangement that is likely to lead to strong fluid-mediated interactions.

To study these interactions, we arrange a tandem pair of identical flags and vary the gap between the two. The gap  $G$  is the streamwise distance between the tail end of the leading flag and the flagpole of the following flag, or, equivalently, the distance between flagpoles less one body length  $L$ . The instantaneous form of the two filaments for nondimensional gap  $G/L = 0$  and the corresponding wake structure are captured in the photograph of Fig. 1(a). The two bodies assume the same frequency of flapping but take on different amplitudes, as revealed in the long-time exposure photograph of Fig. 1(b). For  $G/L = 0$ , the flags flap out of phase. At a greater separation of  $G/L = 0.6$ ,

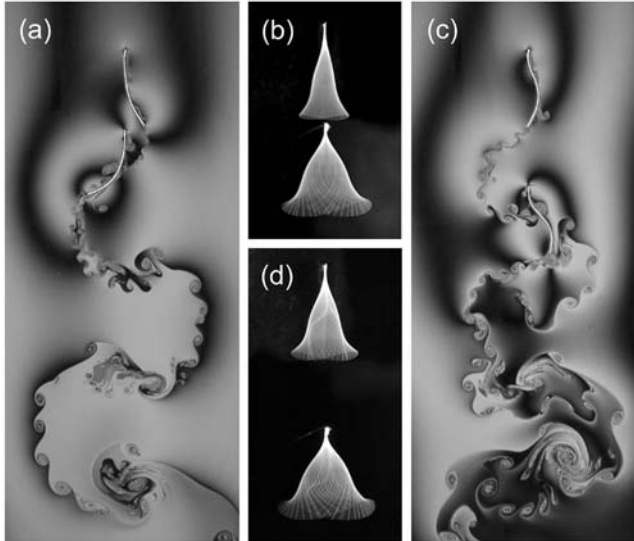


FIG. 1. A pair of tandem flags undulate in a downward flowing soap film. (a),(c) At different separations, the instantaneous flow patterns around the flapping flags are visualized by thin-film interferometry. Separation  $G$  is measured from the end of the leader to the head of its follower. (b),(d) The periodic motion is captured by long-time exposure photographs. When the gap is zero ( $G/L = 0$ ), the wakes of the two flags are united into a coherent structure (a), and the flags take on different lateral amplitudes (b), smaller for the leader than the follower. At a larger separation ( $G/L = 0.6$ ), the flag wakes disentangle (c) and both amplitudes increase (d).

flapping amplitudes increase [Fig. 1(d)], the phase difference is nearly zero, and the wake structure [Fig. 1(c)] becomes more complex. We found that the tandem flags synchronize even when separated by several body lengths. This correlated motion suggests that the fluid-coupled interactions may also lead to altered forces that persist over long lengths.

We first compare the drag force for each member of the pair. The individual flag forces as a function of gap are shown in Fig. 2. In surprising opposition to static objects, we find that the leader always suffers *less* drag than the downstream body. In particular, the leading body experiences less drag than that of an isolated flag  $D_0$ , including a reduction to half at  $G/L = 0$ . The downstream body has drag greater than  $D_0$ . Defying intuition based on fixed bodies, flexible flags experience *inverted drafting*, in which flapping in front reduces fluid forces.

How does this anomalous allotment of drag arise? One might conceive of a scenario in which each flag modifies its surrounding flow *and also* in turn induces changes in the form of motion of the other flag. The latter effect is not possible for rigid objects and is thus a good candidate to explain our new results. Indeed, the flags do have altered dynamics in the aggregate. In particular, measurements of frequency and amplitude reveal two consistent observations. First, the frequency, though somewhat different from

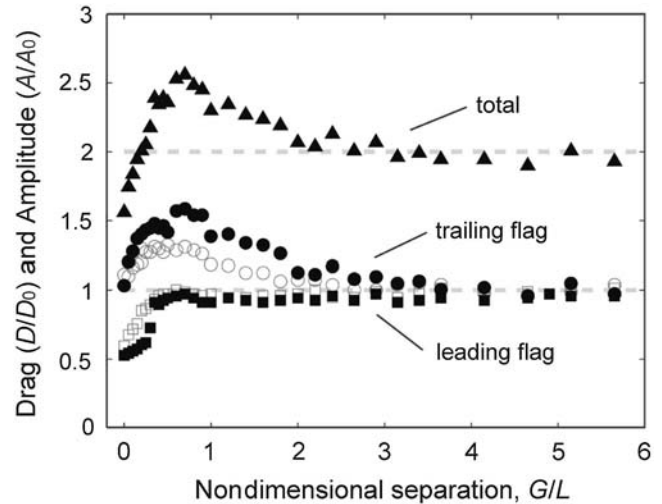


FIG. 2. Normalized streamwise fluid force  $D/D_0$  and lateral amplitude  $A/A_0$  for two tandem flags as a function of nondimensional gap  $G/L$ . The leader (solid squares) experiences a lower drag than the follower (solid circles). In particular, at small separations, the drag on the leader is significantly lower than that of a single, isolated flag ( $D/D_0 < 1$ ). In contrast, the follower experiences a higher than usual drag. Compared to the total drag on two independent flags (the upper dashed line), the total drag for the pair (solid triangles), which is the sum of the data from two lower curves, is reduced for small gaps and amplified at larger gaps. The normalized flapping amplitudes for the leader (open squares) and follower (open circles) each shows a trend similar to the respective drag dependence on gap.

that of a lone flag, is the same for each flag as long as the two were placed within six body lengths of one another. Second, we find that amplitude is strongly dependent on position and is smaller for the leader than the follower, as is evident from the amplitude measurements of Fig. 2 (open symbols).

To understand how these changes in the form of the flapping motion contribute to the drag force, we consider the drag on an object of fixed shape and then estimate how this force depends on the ever-changing shape. For high Reynolds number steady flows, the form drag [18] on a static body is  $C_D \rho U^2 S/2$ . Here,  $\rho$  is the fluid density,  $S$  the area the object presents to the flow, and  $C_D$  the drag coefficient, a shape-dependent parameter of order unity. Thus, the effect of shape appears both through the area  $S$  and the coefficient  $C_D$ . The area  $S$  can be approximated as the product of the film thickness  $d$  and flapping amplitude  $A$ , so that  $S \sim dA$ . If the drag coefficient is relatively constant, then drag scales as the amplitude. Indeed, for both flags, the drag force does appear correlated with the flapping amplitude (Fig. 2). Thus, for tandem flags, inverted drafting reveals itself in the small amplitude for the leading body and considerably broader envelope for the downstream flag, as in Figs. 1(b) and 1(d).

In fact, inverted drafting and the correlation of drag with amplitude are general features of interacting flags. To

demonstrate this, we fix a flag at the origin and measure the drag on a second, nearby flag at varying relative locations. Here, position indicates the displacement between flagpoles. Figure 3(a) shows the map of drag on the flag that is displaced laterally and streamwise, and Fig. 3(b) shows the amplitude map. By comparison, the amplitude of flapping is seen to correlate quite well with the fluid force experienced. The quantity drag/amplitude yields the nearly uniform map of Fig. 3(c), and thus amplitude seems to determine the streamwise drag. Further, inverted drafting is robust to lateral displacement of the flags. Specifically, significant drag reduction for the leader extends upstream over one body length from the origin and more than a half body length to each side [Fig. 3(a)]. The region of higher drag persists four flag lengths downstream and nearly one body length laterally. Taken together, these results indicate that inverted drafting is not sensitive to the exact alignment of the flags but is a result of a broad change in the near-body flow field of the flags.

We then ask how the flow field induces changes in the amplitude of flapping. We first address how the amplitude of the leading flag is reduced, considering the introduction of its downstream neighbor and the flow-flag interaction.

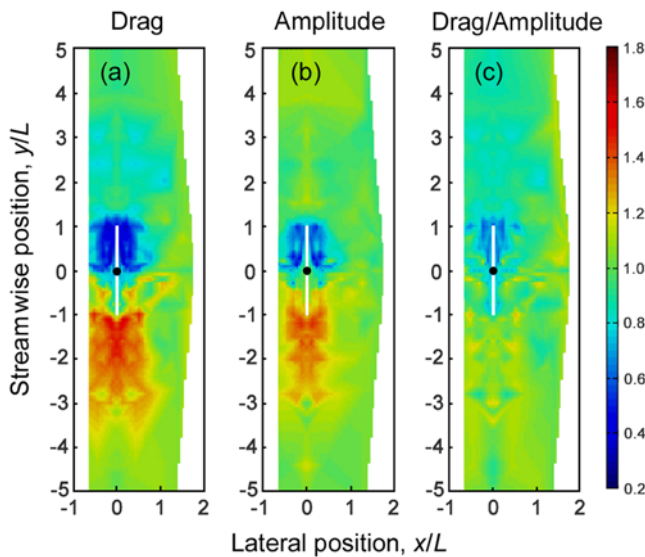


FIG. 3 (color online). Two-dimensional maps of the drag and amplitude for two flapping flags. Here, the head of one flag is fixed at the origin (black dot) while the second flag is displaced laterally and streamwise. The streamwise drag (a) and the flapping amplitude (b) are measured at 338 locations for the second flag. The force map (a) shows a robust drag reduction for the second flag when placed upstream of the first. The second flag takes on, over a greater area, an increased drag when placed downstream. There is a strong correlation of the flapping amplitude (b) with the drag (a), as indicated by the more uniform map of drag normalized by amplitude (c). Data are omitted (white strip) for cases in which the flags collide. All maps are normalized by the corresponding values for an isolated flag at the same flow conditions.

Because the downstream flag is held fixed by a flagpole, this follower serves to suppress lateral flow near the trailing end of the leader. The flapping of the leading flag is thus indirectly confined, because its free end can be viewed as part of the flow structure, namely, a concentrated vortex sheet [13,14]. As a result, the leader presents a smaller cross-sectional area to the oncoming flow, which in turn yields a drag that is less than that on an isolated flag. Here, the role played by the downstream flag is similar to that of a wake splitter, a longitudinal plate that can reduce drag on an upstream body [19,20].

The follower, on the other hand, undulates in the oscillating wake of the leader. Because the flags assume identical frequencies, the following flag flaps at the same frequency at which its oncoming flow oscillates. This resonance effect thus drives the lateral amplitude to increase, resulting in higher drag for the follower.

Though the follower bears the greater drag burden, the pair as a whole can have a drag reduction or increase. This again differs from rigid objects: studies on tandem cylinders reveal that the total drag is always less than that of two independent bodies [4]. For flags of gaps  $G/L < 0.2$ , the total force on the pair is less than that of two independent flags, but at greater separations ( $0.2 < G/L < 3$ ) drag is considerably amplified (triangles in Fig. 2). The flow visualization of Figs. 1(a) and 1(c) offers additional clues to the underpinning of the drag reduction for the aggregate. When tandem flags are close enough to yield drag reduction, the wake of the pair is united into a sinuous, narrow ribbon of vortices that resembles the wake of a single flag [10]. For larger separations, the wake disentangles and widens, and the drag reduction for the pair vanishes. Here the wake width serves as an indicator of drag, which is equal to the rate of removal of fluid momentum.

The lowest drag configuration for both the leading flag and the pair occurs off center at  $(x/L, y/L) = (\pm 0.1, 0.3)$ , for which the leader has normalized drag of 0.25 and the follower 0.70. Thus, the pair has total drag that is about equal to that on a single, lone flag. Interestingly, the two flags are close and flap in phase, undulating like a single flag.

Having investigated pairs of flags, we generalize to larger tandem aggregates. In Fig. 4(a), we show six serial flags at  $G/L = 0$  photographed under strobe lighting. In this larger aggregate, all bodies flap at the same frequency but assume different phases and amplitudes. Across different separation distances, the leading flag enjoys a lower drag than its downstream neighbor, as detailed in Figs. 4(b) and 4(c). That inverted drafting persists in these larger groupings suggests that our rationalization of the two-flag problem offers general insight into the forces on interacting, passively deformable bodies.

It is unknown whether inverted drafting appears in interactions of *actively* flapping bodies, as schooling fish. Though a flag flaps passively and a swimming fish undulates by muscular activation, both motions involve the



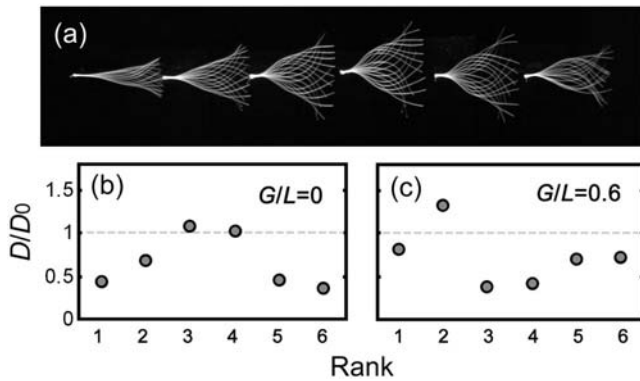


FIG. 4. Drag measurements of six tandem flags. (a) Photograph of flapping motion for  $G/L = 0$ , where flow is from left to right. (b),(c) Drag for each individual flag is shown for two separations. The first flag enjoys a drag reduction compared to the second, meaning inverted drafting persists in these larger aggregates. The group experiences a drag reduction as a whole, as most flags have drag less than  $D_0$ .

interplay of fluid forcing, elasticity, and inertia [10,21,22]. In fact, fish do make use of passive hydrodynamics in unsteady flows for energetic advantage [23]. Presently, nearly all rationalizations of grouping energetics of flapping animals assume no advantage for leading members [5,6]. Such models do not consider any modified dynamics in an aggregate; the motion of a fish, for example, is assumed to be the same in a school as it is during isolated swimming. However, it is known that the form of flapping motion varies according to position in a school and flock [6]. In light of our findings, the altered collective dynamics may lead to dramatic deviations from these models that simply superpose isolated locomotors.

Another class of models describes collective behavior of moving organisms by considering forces of interactions between individuals [24]. Often without explicit reference to an underlying mechanism, some models combine a short-range repulsive force and longer-range attraction, yielding an equilibrium separating distance between members of an aggregate. For tandem flags, inverted drafting directly suggests hydrodynamic repulsion between flags. Because the follower has higher drag, the pair will tend to separate further. This is unlike rigid objects, for which conventional drafting leads to attraction. Thus, the individual forces on tandem flags are such that aggregation is unstable. Notice also that these forces are so arranged as to hinder a flapping “predator” flag in its pursuit of “prey,” the leader. In this case, it is better to be chased than to chase.

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