

$$7. F(x) = \sqrt[4]{1+2x+x^3} = (1+2x+x^3)^{1/4} \Rightarrow$$

$$\begin{aligned} F'(x) &= \frac{1}{4}(1+2x+x^3)^{-3/4} \cdot \frac{d}{dx}(1+2x+x^3) = \frac{1}{4(1+2x+x^3)^{3/4}} \cdot (2+3x^2) \\ &= \frac{2+3x^2}{4(1+2x+x^3)^{3/4}} = \frac{2+3x^2}{4\sqrt[4]{(1+2x+x^3)^3}} \end{aligned}$$

$$17. y = (2x-5)^4(8x^2-5)^{-3} \Rightarrow$$

$$\begin{aligned} y' &= 4(2x-5)^3(2)(8x^2-5)^{-3} + (2x-5)^4(-3)(8x^2-5)^{-4}(16x) \\ &= 8(2x-5)^3(8x^2-5)^{-3} - 48x(2x-5)^4(8x^2-5)^{-4} \end{aligned}$$

$$[\text{This simplifies to } 8(2x-5)^3(8x^2-5)^{-4}(-4x^2+30x-5).]$$

$$21. y = \sin(x \cos x) \Rightarrow y' = \cos(x \cos x) \cdot [x(-\sin x) + \cos x \cdot 1] = (\cos x - x \sin x) \cos(x \cos x)$$

$$30. y = x \sin \frac{1}{x} \Rightarrow y' = \sin \frac{1}{x} + x \cos \frac{1}{x} \left(-\frac{1}{x^2}\right) = \sin \frac{1}{x} - \frac{1}{x} \cos \frac{1}{x}$$

$$36. y = \sqrt{x + \sqrt{x + \sqrt{x}}} \Rightarrow y' = \frac{1}{2} \left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2} \left[1 + \frac{1}{2}(x + \sqrt{x})^{-1/2} \left(1 + \frac{1}{2}x^{-1/2}\right)\right]$$

$$37. y = \sin(\tan \sqrt{\sin x}) \Rightarrow$$

$$\begin{aligned} y' &= \cos(\tan \sqrt{\sin x}) \cdot \frac{d}{dx}(\tan \sqrt{\sin x}) = \cos(\tan \sqrt{\sin x}) \sec^2 \sqrt{\sin x} \cdot \frac{d}{dx}(\sin x)^{1/2} \\ &= \cos(\tan \sqrt{\sin x}) \sec^2 \sqrt{\sin x} \cdot \frac{1}{2}(\sin x)^{-1/2} \cdot \cos x = \cos(\tan \sqrt{\sin x}) \left(\sec^2 \sqrt{\sin x}\right) \left(\frac{1}{2\sqrt{\sin x}}\right) (\cos x) \end{aligned}$$

$$38. y = \sqrt{\cos(\sin^2 x)} \Rightarrow y' = \frac{1}{2}(\cos(\sin^2 x))^{-1/2} [-\sin(\sin^2 x)](2 \sin x \cos x) = -\frac{\sin(\sin^2 x) \sin x \cos x}{\sqrt{\cos(\sin^2 x)}}$$

$$55. r(x) = f(g(h(x))) \Rightarrow r'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x), \text{ so}$$

$$r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) = f'(g(2)) \cdot g'(2) \cdot 4 = f'(3) \cdot 5 \cdot 4 = 6 \cdot 5 \cdot 4 = 120$$

$$60. (a) s = A \cos(\omega t + \delta) \Rightarrow \text{velocity} = s' = -\omega A \sin(\omega t + \delta).$$

$$(b) \text{ If } A \neq 0 \text{ and } \omega \neq 0, \text{ then } s' = 0 \Leftrightarrow \sin(\omega t + \delta) = 0 \Leftrightarrow \omega t + \delta = n\pi \Leftrightarrow t = \frac{n\pi - \delta}{\omega}, n \text{ an integer.}$$

$$\begin{aligned}
 69. \quad \frac{d^2 y}{dx^2} &= \frac{d}{dx} \left(\frac{dy}{dx} \right) && \text{[Leibniz notation for the second derivative]} \\
 &= \frac{d}{dx} \left(\frac{dy}{du} \frac{du}{dx} \right) && \text{[Chain Rule]} \\
 &= \frac{dy}{du} \cdot \frac{d}{dx} \left(\frac{du}{dx} \right) + \frac{du}{dx} \cdot \frac{d}{dx} \left(\frac{dy}{du} \right) && \text{[Product Rule]} \\
 &= \frac{dy}{du} \cdot \frac{d^2 u}{dx^2} + \frac{du}{dx} \cdot \frac{d}{du} \left(\frac{dy}{du} \right) \cdot \frac{du}{dx} && \text{[} dy/du \text{ is a function of } u \text{]} \\
 &= \frac{dy}{du} \frac{d^2 u}{dx^2} + \frac{d^2 y}{du^2} \left(\frac{du}{dx} \right)^2
 \end{aligned}$$

Or: Using function notation for $y = f(u)$ and $u = g(x)$, we have $y = f(g(x))$, so

$$y' = f'(g(x)) \cdot g'(x) \quad \text{[by the Chain Rule]} \quad \Rightarrow$$

$$(y')' = [f'(g(x)) \cdot g'(x)]' = f'(g(x)) \cdot g''(x) + g'(x) \cdot f''(g(x)) \cdot g'(x) = f'(g(x)) \cdot g''(x) + f''(g(x)) \cdot [g'(x)]^2.$$