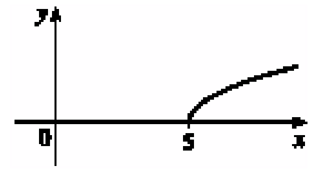
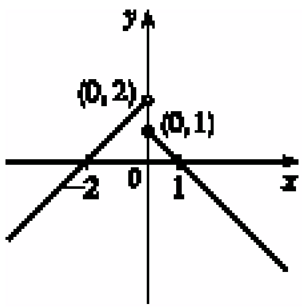
33. $g(x) = \sqrt{x-5}$ is defined when $x - 5 \geq 0$ or $x \geq 5$, so the domain is $[5, \infty)$.
Since $y = \sqrt{x-5} \quad \Rightarrow \quad y^2 = x - 5 \quad \Rightarrow \quad x = y^2 + 5$, we see that g is the top half of a parabola.



37. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$

The domain is $\mathbb{R}$.



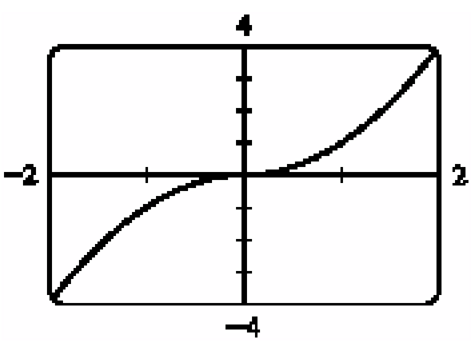
42. The slope of this line segment is $\dfrac{3-(-2)}{6-(-3)} = \dfrac{5}{9}$, so an equation is $y + 2 = \frac{5}{9}(x+3)$.

The function is $f(x) = \frac{5}{9}x - \frac{1}{3}$, $-3 \leq x \leq 6$.

60. $f(x) = x\,|x|$.

$$\begin{aligned} f(-x) &= (-x)\,|-x| = (-x)\,|x| = -(x\,|x|) \\ &= -f(x) \end{aligned}$$

So f is an odd function.



6. (a) For $T = 0.02t + 8.50$, the slope is 0.02, which means that the average surface temperature of the world is increasing at a rate of 0.02°C per year. The T-intercept is 8.50, which represents the average surface temperature in °C in the year 1900.

(b) $t = 2100 - 1900 = 200 \quad \Rightarrow \quad T = 0.02(200) + 8.50 = 12.50°\text{C}$

16. (a) To obtain the graph of $y = 5f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5.

(b) To obtain the graph of $y = f(x - 5)$ from the graph of $y = f(x)$, shift the graph 5 units to the right.

(c) To obtain the graph of $y = -f(x)$ from the graph of $y = f(x)$, reflect the graph about the x-axis.

(d) To obtain the graph of $y = -5f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5 and reflect it about the x-axis.

(e) To obtain the graph of $y = f(5x)$ from the graph of $y = f(x)$, shrink the graph horizontally by a factor of 5.

(f) To obtain the graph of $y = 5f(x) - 3$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5 and shift it 3 units downward.

37. $f(x) = x^2 - 1$, $D = \mathbb{R}$; $g(x) = 2x + 1$, $D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2 - 1 = (4x^2 + 4x + 1) - 1 = 4x^2 + 4x$, $D = \mathbb{R}$.

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 - 1) = 2(x^2 - 1) + 1 = (2x^2 - 2) + 1 = 2x^2 - 1$, $D = \mathbb{R}$.

(c) $(f \circ f)(x) = f(f(x)) = f(x^2 - 1) = (x^2 - 1)^2 - 1 = (x^4 - 2x^2 + 1) - 1 = x^4 - 2x^2$, $D = \mathbb{R}$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x + 1) = 2(2x + 1) + 1 = (4x + 2) + 1 = 4x + 3$, $D = \mathbb{R}$.

41. $f(x) = x + \dfrac{1}{x}, \quad D = \{x \mid x \neq 0\}; \quad g(x) = \dfrac{x+1}{x+2}, \quad D = \{x \mid x \neq -2\}.$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x+1}{x+2}\right) = \dfrac{x+1}{x+2} + \dfrac{1}{\frac{x+1}{x+2}} = \dfrac{x+1}{x+2} + \dfrac{x+2}{x+1}$

$$= \frac{(x+1)(x+1)+(x+2)(x+2)}{(x+2)(x+1)} = \frac{(x^2+2x+1)+(x^2+4x+4)}{(x+2)(x+1)} = \frac{2x^2+6x+5}{(x+2)(x+1)}$$

Since $g(x)$ is not defined for $x = -2$ and $f(g(x))$ is not defined for $x = -2$ and $x = -1$, the domain of $(f \circ g)(x)$ is $D = \{x \mid x \neq -2, -1\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(x + \dfrac{1}{x}\right) = \dfrac{\left(x+\frac{1}{x}\right)+1}{\left(x+\frac{1}{x}\right)+2} = \dfrac{\frac{x^2+1+x}{x}}{\frac{x^2+1+2x}{x}} = \dfrac{x^2+x+1}{x^2+2x+1} = \dfrac{x^2+x+1}{(x+1)^2}$

Since $f(x)$ is not defined for $x = 0$ and $g(f(x))$ is not defined for $x = -1$, the domain of $(g \circ f)(x)$ is $D = \{x \mid x \neq -1, 0\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(x + \dfrac{1}{x}\right) = \left(x + \dfrac{1}{x}\right) + \dfrac{1}{x + \frac{1}{x}} = x + \dfrac{1}{x} + \dfrac{1}{\frac{x^2+1}{x}} = x + \dfrac{1}{x} + \dfrac{x}{x^2+1}$

$$= \frac{x(x)(x^2+1)+1\,(x^2+1)+x(x)}{x(x^2+1)} = \frac{x^4+x^2+x^2+1+x^2}{x(x^2+1)}$$

$$= \frac{x^4+3x^2+1}{x(x^2+1)}, \quad D = \{x \mid x \neq 0\}.$$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{x+1}{x+2}\right) = \dfrac{\frac{x+1}{x+2}+1}{\frac{x+1}{x+2}+2} = \dfrac{\frac{x+1+1(x+2)}{x+2}}{\frac{x+1+2(x+2)}{x+2}} = \dfrac{x+1+x+2}{x+1+2x+4} = \dfrac{2x+3}{3x+5}$

Since $g(x)$ is not defined for $x = -2$ and $g(g(x))$ is not defined for $x = -\frac{5}{3}$, the domain of $(g \circ g)(x)$ is $D = \left\{x \mid x \neq -2, -\frac{5}{3}\right\}$.

43. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x+3)) = f((x+3)^2+2) = f(x^2+6x+11) = \sqrt{(x^2+6x+11)-1} = \sqrt{x^2+6x+10}$

46. Let $g(x) = \sqrt{x}$ and $f(x) = \sin x$. Then $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right) = \sin\left(\sqrt{x}\right) = F(x)$.

47. Let $g(t) = \cos t$ and $f(t) = \sqrt{t}$. Then $(f \circ g)(t) = f(g(t)) = f(\cos t) = \sqrt{\cos t} = u(t)$.

50. Let $h(x) = |x|$, $g(x) = 2 + x$, and $f(x) = \sqrt[8]{x}$. Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f\,(2+|x|) = \sqrt[8]{2+|x|} = H(x).$$

52. (a) $f(g(1)) = f(6) = 5$

(b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$

(d) $g(g(1)) = g(6) = 3$

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

12. For $f(x) = \dfrac{x^2 - 2x}{x^2 - x - 2}$:

x	$f(x)$
0	0
-0.5	-1
-0.9	-9
-0.95	-19
-0.99	-99
-0.999	-999

x	$f(x)$
-2	2
-1.5	3
-1.1	11
-1.01	101
-1.001	1001

It appears that $\lim\limits_{x \to -1} \dfrac{x^2 - 2x}{x^2 - x - 2}$ *does not exist* since

$f(x) \to \infty$ as $x \to -1^-$ and $f(x) \to -\infty$ as $x \to -1^+$.

27. (a) $A = \pi r^2$ and $A = 1000 \text{ cm}^2 \;\Rightarrow\; \pi r^2 = 1000 \;\Rightarrow\; r^2 = \frac{1000}{\pi} \;\Rightarrow\; r = \sqrt{\frac{1000}{\pi}} \quad [r > 0] \quad \approx 17.8412$ cm.

(b) $|A - 1000| \le 5 \;\Rightarrow\; -5 \le \pi r^2 - 1000 \le 5 \;\Rightarrow\; 1000 - 5 \le \pi r^2 \le 1000 + 5 \;\Rightarrow$

$\sqrt{\frac{995}{\pi}} \le r \le \sqrt{\frac{1005}{\pi}} \;\Rightarrow\; 17.7966 \le r \le 17.8858$. $\sqrt{\frac{1000}{\pi}} - \sqrt{\frac{995}{\pi}} \approx 0.04466$ and $\sqrt{\frac{1005}{\pi}} - \sqrt{\frac{1000}{\pi}} \approx 0.04455$.

So if the machinist gets the radius within 0.0445 cm of 17.8412, the area will be within 5 cm^2 of 1000.

(c) x is the radius, $f(x)$ is the area, a is the target radius given in part (a), L is the target area (1000), ε is the tolerance in the area (5), and δ is the tolerance in the radius given in part (b).

39. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x - 0| < \delta$, then $|x^2 - 0| < \varepsilon \;\Leftrightarrow\; x^2 < \varepsilon \;\Leftrightarrow\; |x| < \sqrt{\varepsilon}$. Take $\delta = \sqrt{\varepsilon}$. Then $0 < |x - 0| < \delta \;\Rightarrow\; |x^2 - 0| < \varepsilon$. Thus, $\lim\limits_{x \to 0} x^2 = 0$ by the definition of a limit.

41. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x - 0| < \delta$, then $\big||x| - 0\big| < \varepsilon$. But $\big||x|\big| = |x|$. So this is true if we pick $\delta = \varepsilon$. Thus, $\lim\limits_{x \to 0} |x| = 0$ by the definition of a limit.

1. (a) $\lim_{x\to a}[f(x)+h(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} h(x) = -3+8=5$

(b) $\lim_{x\to a}[f(x)]^2 = \left[\lim_{x\to a} f(x)\right]^2 = (-3)^2 = 9$

(c) $\lim_{x\to a}\sqrt[3]{h(x)} = \sqrt[3]{\lim_{x\to a} h(x)} = \sqrt[3]{8} = 2$

(d) $\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{\lim_{x\to a} f(x)} = \frac{1}{-3} = -\frac{1}{3}$

(e) $\lim_{x\to a}\frac{f(x)}{h(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} h(x)} = \frac{-3}{8} = -\frac{3}{8}$

(f) $\lim_{x\to a}\frac{g(x)}{f(x)} = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} f(x)} = \frac{0}{-3} = 0$

(g) The limit does not exist, since $\lim_{x\to a} g(x) = 0$ but $\lim_{x\to a} f(x) \neq 0$.

(h) $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)} = \frac{2\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)} = \frac{2(-3)}{8-(-3)} = -\frac{6}{11}$

2. (a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2+0=2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0\cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3\cdot 2 = 16$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

17. $\lim_{h\to 0}\frac{(4+h)^2-16}{h} = \lim_{h\to 0}\frac{(16+8h+h^2)-16}{h} = \lim_{h\to 0}\frac{8h+h^2}{h} = \lim_{h\to 0}\frac{h(8+h)}{h} = \lim_{h\to 0}(8+h) = 8+0=8$

18. $\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}\cdot\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0}\frac{(1+h)-1}{h\left(\sqrt{1+h}+1\right)} = \lim_{h\to 0}\frac{h}{h\left(\sqrt{1+h}+1\right)}$

$= \lim_{h\to 0}\frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

20. $\lim_{x\to -1}\frac{x^2+2x+1}{x^4-1} = \lim_{x\to -1}\frac{(x+1)^2}{(x^2+1)(x^2-1)} = \lim_{x\to -1}\frac{(x+1)^2}{(x^2+1)(x+1)(x-1)} = \lim_{x\to -1}\frac{x+1}{(x^2+1)(x-1)} = \frac{0}{2(-2)} = 0$

31. $-1 \leq \cos(2/x) \leq 1 \quad\Rightarrow\quad -x^4 \leq x^4\cos(2/x) \leq x^4$. Since $\lim_{x\to 0}\left(-x^4\right) = 0$ and $\lim_{x\to 0} x^4 = 0$, we have

$\lim_{x\to 0}\left[x^4\cos(2/x)\right] = 0$ by the Squeeze Theorem.

43. $\displaystyle\lim_{x\to0}\frac{\sin 3x}{x} = \lim_{x\to0}\frac{3\sin 3x}{3x}$ [multiply numerator and denominator by 3]

$\displaystyle = 3\lim_{3x\to0}\frac{\sin 3x}{3x}$ [as $x\to0$, $3x\to0$]

$\displaystyle = 3\lim_{\theta\to0}\frac{\sin\theta}{\theta}$ [let $\theta = 3x$]

$= 3(1)$ [Equation 2]

$= 3$

45. $$\lim_{t\to0}\frac{\tan 6t}{\sin 2t} = \lim_{t\to0}\left(\frac{\sin 6t}{t}\cdot\frac{1}{\cos 6t}\cdot\frac{t}{\sin 2t}\right) = \lim_{t\to0}\frac{6\sin 6t}{6t}\cdot\lim_{t\to0}\frac{1}{\cos 6t}\cdot\lim_{t\to0}\frac{2t}{2\sin 2t}$$

$$= 6\lim_{t\to0}\frac{\sin 6t}{6t}\cdot\lim_{t\to0}\frac{1}{\cos 6t}\cdot\frac{1}{2}\lim_{t\to0}\frac{2t}{\sin 2t} = 6(1)\cdot\frac{1}{1}\cdot\frac{1}{2}(1) = 3$$