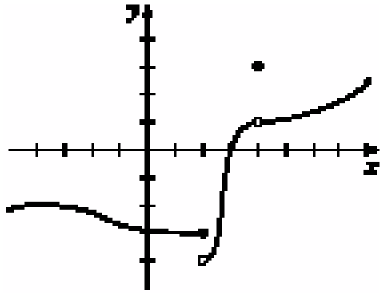
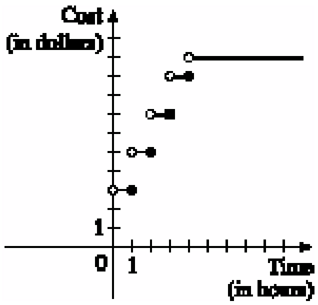
3. (a) The following are the numbers at which f is discontinuous and the type of discontinuity at that number: -4 (removable), -2 (jump), 2 (jump), 4 (infinite).

(b) f is continuous from the left at -2 since $\lim_{x\to -2^-} f(x) = f(-2)$. f is continuous from the right at 2 and 4 since $\lim_{x\to 2^+} f(x) = f(2)$ and $\lim_{x\to 4^+} f(x) = f(4)$. It is continuous from neither side at -4 since $f(-4)$ is undefined.

6.



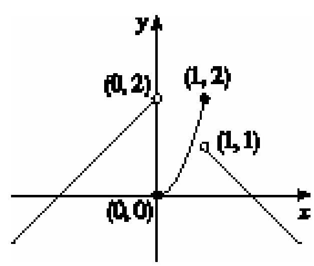
7. (a)



(b) There are discontinuities at times $t = 1, 2, 3,$ and 4. A person parking in the lot would want to keep in mind that the charge will jump at the beginning of each hour.

21. By Theorem 6, the root function $\sqrt{x}$ and the trigonometric function $\sin x$ are continuous on their domains, $[0,\infty)$ and $(-\infty,\infty)$, respectively. Thus, the product $F(x) = \sqrt{x}\sin x$ is continuous on the intersection of those domains, $[0,\infty)$, by part 4 of Theorem 4.

29. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 2 - x & \text{if } x > 1 \end{cases}$



f is continuous on $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$ since on each of these intervals it is a polynomial. Now $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x+2) = 2$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x^2 = 0$, so f is discontinuous at 0. Since $f(0) = 0$, f is continuous from the right at 0. Also $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x^2 = 2$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2-x) = 1$, so f is discontinuous at 1. Since $f(1) = 2$, f is continuous from the left at 1.

31. $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$

f is continuous on $(-\infty, 2)$ and $(2, \infty)$. Now $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left(cx^2 + 2x\right) = 4c + 4$ and $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(x^3 - cx\right) = 8 - 2c$. So f is continuous $\Leftrightarrow$ $4c + 4 = 8 - 2c$ $\Leftrightarrow$ $6c = 4$ $\Leftrightarrow$ $c = \frac{2}{3}$. Thus, for f to be continuous on $(-\infty, \infty)$, $c = \frac{2}{3}$.

35. $f(x) = x^2 + 10 \sin x$ is continuous on the interval $[31, 32]$, $f(31) \approx 957$, and $f(32) \approx 1030$. Since $957 < 1000 < 1030$, there is a number c in $(31, 32)$ such that $f(c) = 1000$ by the Intermediate Value Theorem.

2. (a) $\lim_{x \to \infty} g(x) = 2$ (b) $\lim_{x \to -\infty} g(x) = -2$ (c) $\lim_{x \to 3} g(x) = \infty$

(d) $\lim_{x \to 0} g(x) = -\infty$ (e) $\lim_{x \to -2^+} g(x) = -\infty$ (f) Vertical: $x = -2$, $x = 0$, $x = 3$; Horizontal: $y = -2$, $y = 2$

19. Divide both the numerator and denominator by x^3 (the highest power of x that occurs in the denominator).

$$\lim_{x \to \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4} = \lim_{x \to \infty} \frac{\dfrac{x^3 + 5x}{x^3}}{\dfrac{2x^3 - x^2 + 4}{x^3}} = \lim_{x \to \infty} \frac{1 + \dfrac{5}{x^2}}{2 - \dfrac{1}{x} + \dfrac{4}{x^3}} = \frac{\lim_{x \to \infty} \left(1 + \dfrac{5}{x^2}\right)}{\lim_{x \to \infty} \left(2 - \dfrac{1}{x} + \dfrac{4}{x^3}\right)}$$

$$= \frac{\lim_{x \to \infty} 1 + 5 \lim_{x \to \infty} \dfrac{1}{x^2}}{\lim_{x \to \infty} 2 - \lim_{x \to \infty} \dfrac{1}{x} + 4 \lim_{x \to \infty} \dfrac{1}{x^3}} = \frac{1 + 5(0)}{2 - 0 + 4(0)} = \frac{1}{2}$$

23.
$$\lim_{x \to \infty} \left(\sqrt{9x^2 + x} - 3x\right) = \lim_{x \to \infty} \frac{\left(\sqrt{9x^2 + x} - 3x\right)\left(\sqrt{9x^2 + x} + 3x\right)}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{\left(\sqrt{9x^2 + x}\right)^2 - (3x)^2}{\sqrt{9x^2 + x} + 3x}$$

$$= \lim_{x \to \infty} \frac{(9x^2 + x) - 9x^2}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{x}{\sqrt{9x^2 + x} + 3x} \cdot \frac{1/x}{1/x}$$

$$= \lim_{x \to \infty} \frac{x/x}{\sqrt{9x^2/x^2 + x/x^2} + 3x/x} = \lim_{x \to \infty} \frac{1}{\sqrt{9 + 1/x} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{3 + 3} = \frac{1}{6}$$

26. Since $0 \leq \sin^2 x \leq 1$, we have $0 \leq \dfrac{\sin^2 x}{x^2} \leq \dfrac{1}{x^2}$. Now $\lim_{x \to \infty} 0 = 0$ and $\lim_{x \to \infty} \dfrac{1}{x^2} = 0$, so by the Squeeze Theorem, $\lim_{x \to \infty} \dfrac{\sin^2 x}{x^2} = 0$.

27. $\lim_{x \to \infty} \left(x - \sqrt{x}\,\right) = \lim_{x \to \infty} \sqrt{x}\left(\sqrt{x} - 1\right) = \infty$ since $\sqrt{x} \to \infty$ and $\sqrt{x} - 1 \to \infty$ as $x \to \infty$.

31. $\displaystyle\lim_{x\to\infty}\frac{x+x^3+x^5}{1-x^2+x^4}=\lim_{x\to\infty}\frac{(x+x^3+x^5)/x^4}{(1-x^2+x^4)/x^4}$ [divide by the highest power of x in the denominator]

$$=\lim_{x\to\infty}\frac{1/x^3+1/x+x}{1/x^4-1/x^2+1}=\infty$$

because $(1/x^3+1/x+x)\to\infty$ and $(1/x^4-1/x^2+1)\to 1$ as $x\to\infty$.

38. Since the function has vertical asymptotes $x=1$ and $x=3$, the denominator of the rational function we are looking for must have factors $(x-1)$ and $(x-3)$. Because the horizontal asymptote is $y=1$, the degree of the numerator must equal the degree of the denominator, and the ratio of the leading coefficients must be 1. One possibility is $f(x)=\dfrac{x^2}{(x-1)(x-3)}$.