3. Using (1) with $f(x) = \dfrac{x-1}{x-2}$ and $P(3, 2)$,

$$m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{x\to 3} \frac{\dfrac{x-1}{x-2} - 2}{x-3} = \lim_{x\to 3} \frac{\dfrac{x-1-2(x-2)}{x-2}}{x-3} = \lim_{x\to 3} \frac{3-x}{(x-2)(x-3)} = \lim_{x\to 3} \frac{-1}{x-2} = \frac{-1}{1} = -1.$$

Tangent line: $y - 2 = -1(x-3) \quad \Leftrightarrow \quad y - 2 = -x + 3 \quad \Leftrightarrow \quad y = -x + 5$

5. Using (1), $m = \displaystyle\lim_{x\to 1} \frac{\sqrt{x} - \sqrt{1}}{x-1} = \lim_{x\to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}.$

Tangent line: $y - 1 = \frac{1}{2}(x-1) \quad \Leftrightarrow \quad y = \frac{1}{2}x + \frac{1}{2}$

12. (a) $v(1) = \displaystyle\lim_{h\to 0} \frac{H(1+h) - H(1)}{h} = \lim_{h\to 0} \frac{\left(58 + 58h - 0.83 - 1.66h - 0.83h^2\right) - 57.17}{h}$

$\qquad = \displaystyle\lim_{h\to 0} (56.34 - 0.83h) = 56.34 \text{ m/s}$

(b) $v(a) = \displaystyle\lim_{h\to 0} \frac{H(a+h) - H(a)}{h} = \lim_{h\to 0} \frac{\left(58a + 58h - 0.83a^2 - 1.66ah - 0.83h^2\right) - \left(58a - 0.83a^2\right)}{h}$

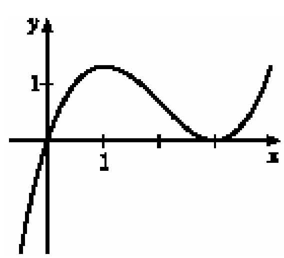
$\qquad = \displaystyle\lim_{h\to 0} (58 - 1.66a - 0.83h) = 58 - 1.66a \text{ m/s}$

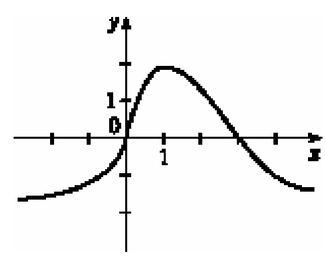
(c) The arrow strikes the moon when the height is 0, that is, $58t - 0.83t^2 = 0 \quad \Leftrightarrow \quad t(58 - 0.83t) = 0 \quad \Leftrightarrow$
$t = \dfrac{58}{0.83} \approx 69.9$ s (since t can't be 0).

(d) Using the time from part (c), $v\left(\dfrac{58}{0.83}\right) = 58 - 1.66\left(\dfrac{58}{0.83}\right) = -58$ m/s. Thus, the arrow will have a velocity of -58 m/s.

15. $g'(0)$ is the only negative value. The slope at $x = 4$ is smaller than the slope at $x = 2$ and both are smaller than the slope at $x = -2$. Thus, $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$.

17. We begin by drawing a curve through the origin with a slope of 3 to satisfy $f(0) = 0$ and $f'(0) = 3$. Since $f'(1) = 0$, we will round off our figure so that there is a horizontal tangent directly over $x = 1$. Last, we make sure that the curve has a slope of -1 as we pass over $x = 2$. Two of the many possibilities are shown.





Note that the answers to Exercises 29 – 34 are not unique.

29. By Definition 4, $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h} = f'(1)$, where $f(x) = x^{10}$ and $a = 1$.

Or: By Definition 4, $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

47. Since $f(x) = x\sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h\sin(1/h)-0}{h} = \lim_{h\to 0} \sin(1/h)$. This limit does not exist since $\sin(1/h)$ takes the values -1 and 1 on any interval containing 0. (Compare with Example 5 in Section 1.3.)

3. (a)$'$ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

(b)$'$ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)$'$ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)$'$ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

19. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left[(x+h)^3 - 3(x+h) + 5\right] - (x^3 - 3x + 5)}{h}$$
$$= \lim_{h\to 0} \frac{\left(x^3 + 3x^2h + 3xh^2 + h^3 - 3x - 3h + 5\right) - \left(x^3 - 3x + 5\right)}{h} = \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 - 3h}{h}$$
$$= \lim_{h\to 0} \frac{h\left(3x^2 + 3xh + h^2 - 3\right)}{h} = \lim_{h\to 0} \left(3x^2 + 3xh + h^2 - 3\right) = 3x^2 - 3$$

Domain of f = domain of $f' = \mathbb{R}$.

21. $$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h}\left[\frac{\sqrt{1+2(x+h)} + \sqrt{1+2x}}{\sqrt{1+2(x+h)} + \sqrt{1+2x}}\right]$$
$$= \lim_{h\to 0} \frac{(1+2x+2h) - (1+2x)}{h\left[\sqrt{1+2(x+h)} + \sqrt{1+2x}\right]} = \lim_{h\to 0} \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}} = \frac{2}{2\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}}$$

Domain of $g = \left[-\frac{1}{2}, \infty\right)$, domain of $g' = \left(-\frac{1}{2}, \infty\right)$.

30. f is not differentiable at $x = -1$, because there is a discontinuity there, and at $x = 2$, because the graph has a corner there.

39. (a) Note that we have factored $x - a$ as the difference of two cubes in the third step.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^{1/3} - a^{1/3}}{x - a} = \lim_{x \to a} \frac{x^{1/3} - a^{1/3}}{(x^{1/3} - a^{1/3})(x^{2/3} + x^{1/3}a^{1/3} + a^{2/3})}$$

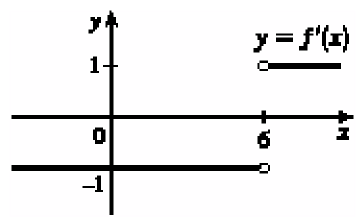
$$= \lim_{x \to a} \frac{1}{x^{2/3} + x^{1/3}a^{1/3} + a^{2/3}} = \frac{1}{3a^{2/3}} \text{ or } \tfrac{1}{3}a^{-2/3}$$

(b) $f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\sqrt[3]{h} - 0}{h} = \lim_{h \to 0} \frac{1}{h^{2/3}}$. This function increases without bound, so the limit does not exist, and therefore $f'(0)$ does not exist.

(c) $\lim_{x \to 0} |f'(x)| = \lim_{x \to 0} \frac{1}{3x^{2/3}} = \infty$ and f is continuous at $x = 0$ (root function), so f has a vertical tangent at $x = 0$.

41. $f(x) = |x - 6| = \begin{cases} x - 6 & \text{if } x - 6 \geq 6 \\ -(x - 6) & \text{if } x - 6 < 0 \end{cases} = \begin{cases} x - 6 & \text{if } x \geq 6 \\ 6 - x & \text{if } x < 6 \end{cases}$

So the right-hand limit is $\lim_{x \to 6^+} \frac{f(x) - f(6)}{x - 6} = \lim_{x \to 6^+} \frac{|x - 6| - 0}{x - 6} = \lim_{x \to 6^+} \frac{x - 6}{x - 6} = \lim_{x \to 6^+} 1 = 1$, and the left-hand limit is $\lim_{x \to 6^-} \frac{f(x) - f(6)}{x - 6} = \lim_{x \to 6^-} \frac{|x - 6| - 0}{x - 6} = \lim_{x \to 6^-} \frac{6 - x}{x - 6} = \lim_{x \to 6^-} (-1) = -1$. Since these limits are not equal, $f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6}$ does not exist and f is not differentiable at 6.



However, a formula for f' is $f'(x) = \begin{cases} 1 & \text{if } x > 6 \\ -1 & \text{if } x < 6 \end{cases}$

Another way of writing the formula is $f'(x) = \frac{x - 6}{|x - 6|}$.