

6. Let $u = \sin \theta$. Then $du = \cos \theta d\theta$, so $\int e^{\sin \theta} \cos \theta d\theta = \int e^u du = e^u + C = e^{\sin \theta} + C$.

20. Let $u = 2\theta$. Then $du = 2 d\theta$ and $d\theta = \frac{1}{2} du$, so $\int \sec 2\theta \tan 2\theta d\theta = \int \sec u \tan u \left(\frac{1}{2} du\right) = \frac{1}{2} \sec u + C = \frac{1}{2} \sec 2\theta + C$.

33. Let $u = 1 + x^2$. Then $du = 2x dx$, so

$$\begin{aligned} \int \frac{1+x}{1+x^2} dx &= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx = \tan^{-1} x + \int \frac{\frac{1}{2} du}{u} = \tan^{-1} x + \frac{1}{2} \ln|u| + C \\ &= \tan^{-1} x + \frac{1}{2} \ln|1+x^2| + C = \tan^{-1} x + \frac{1}{2} \ln(1+x^2) + C \text{ [since } 1+x^2 > 0]. \end{aligned}$$

49. Let $u = \ln x$, so $du = \frac{dx}{x}$. When $x = e$, $u = 1$; when $x = e^4$, $u = 4$.

$$\text{Thus, } \int_e^{e^4} \frac{dx}{x \sqrt{\ln x}} = \int_1^4 u^{-1/2} du = 2 \left[u^{1/2} \right]_1^4 = 2(2 - 1) = 2.$$

56. Let $u = x^2$. Then $du = 2x dx$ and the limits are unchanged ($0^2 = 0$ and $1^2 = 1$), so

$$I = \int_0^1 x \sqrt{1-x^4} dx = \frac{1}{2} \int_0^1 \sqrt{1-u^2} du. \text{ But this integral can be interpreted as the area of a quarter-circle with radius 1.}$$

$$\text{So } I = \frac{1}{2} \cdot \frac{1}{4} (\pi \cdot 1^2) = \frac{1}{8} \pi.$$

63. Let $u = -x$. Then $du = -dx$, so

$$\int_a^b f(-x) dx = \int_{-a}^{-b} f(u)(-du) = \int_{-b}^{-a} f(u) du = \int_{-b}^{-a} f(x) dx.$$

From the diagram, we see that the equality follows from the fact that we are reflecting the graph of f , and the limits of integration, about the y -axis.

