

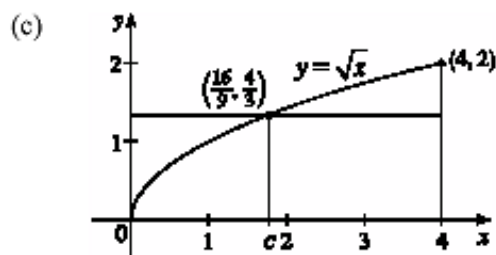
5.  $f(t) = \sqrt{1+2t}$  and  $g(x) = \int_0^x \sqrt{1+2t} dt$ , so by FTC1,  $g'(x) = f(x) = \sqrt{1+2x}$ .

11. Let  $u = \sqrt{x}$ . Then  $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ . Also,  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ , so

$$y' = \frac{d}{dx} \int_3^{\sqrt{x}} \frac{\cos t}{t} dt = \frac{d}{du} \int_3^u \frac{\cos t}{t} dt \cdot \frac{du}{dx} = \frac{\cos u}{u} \cdot \frac{1}{2\sqrt{x}} = \frac{\cos \sqrt{x}}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{\cos \sqrt{x}}{2x}.$$

20. (a)  $f_{\text{ave}} = \frac{1}{4-0} \int_0^4 \sqrt{x} dx = \frac{1}{4} \left[ \frac{2}{3} x^{3/2} \right]_0^4$   
 $= \frac{1}{6} \left[ x^{3/2} \right]_0^4 = \frac{1}{6} [8 - 0] = \frac{4}{3}$

(b)  $f(c) = f_{\text{ave}} \Leftrightarrow \sqrt{c} = \frac{4}{3} \Leftrightarrow c = \frac{16}{9}$



23.  $F(x) = \int_1^x f(t) dt \Rightarrow F'(x) = f(x) = \int_1^{x^2} \frac{\sqrt{1+u^4}}{u} du \left[ \text{since } f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u} du \right] \Rightarrow$   
 $F''(x) = f'(x) = \frac{\sqrt{1+(x^2)^4}}{x^2} \cdot \frac{d}{dx}(x^2) = \frac{\sqrt{1+x^8}}{x^2} \cdot 2x = \frac{2\sqrt{1+x^8}}{x}.$  So  $F''(2) = \sqrt{1+2^8} = \sqrt{257}.$

33. (a) Let  $F(t) = \int_0^t f(s) ds$ . Then, by FTC1,  $F'(t) = f(t)$  = rate of depreciation, so  $F(t)$  represents the loss in value over the interval  $[0, t]$ .

(b)  $C(t) = \frac{1}{t} \left[ A + \int_0^t f(s) ds \right] = \frac{A + F(t)}{t}$  represents the average expenditure per unit of  $t$  during the interval  $[0, t]$ , assuming that there has been only one overhaul during that time period. The company wants to minimize average expenditure.

(c)  $C(t) = \frac{1}{t} \left[ A + \int_0^t f(s) ds \right]$ . Using FTC1, we have  $C'(t) = -\frac{1}{t^2} \left[ A + \int_0^t f(s) ds \right] + \frac{1}{t} f(t).$

$$C'(t) = 0 \Rightarrow t f(t) = A + \int_0^t f(s) ds \Rightarrow f(t) = \frac{1}{t} \left[ A + \int_0^t f(s) ds \right] = C(t).$$