

$$8. y = \sqrt{1+x^3} \Rightarrow \frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} = \frac{1}{2}(1+x^3)^{-1/2}(3x^2) \frac{dx}{dt} = \frac{3x^2}{2\sqrt{1+x^3}} \frac{dx}{dt}. \text{ With } \frac{dy}{dt} = 4 \text{ when } x = 2 \text{ and } y = 3,$$

$$\text{we have } 4 = \frac{3(4)}{2(3)} \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 2 \text{ cm/s.}$$

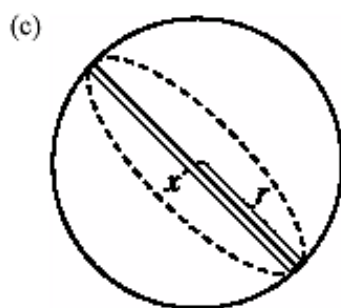
9. (a) Given: the rate of decrease of the surface area is $1 \text{ cm}^2/\text{min}$. If we let t be time (in minutes) and S be the surface area (in cm^2), then we are given that $dS/dt = -1 \text{ cm}^2/\text{s}$.

- (b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10 \text{ cm}$.

- (d) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and

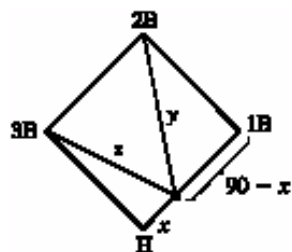
$$S = 4\pi r^2 = 4\pi \left(\frac{1}{2}x\right)^2 = \pi x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \frac{dx}{dt} = 2\pi x \frac{dx}{dt}.$$

- (e) $-1 = \frac{dS}{dt} = 2\pi x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{1}{2\pi x}$. When $x = 10$, $\frac{dx}{dt} = -\frac{1}{20\pi}$. So the rate of decrease is $\frac{1}{20\pi} \text{ cm/min}$.



16. We are given that $\frac{dx}{dt} = 24 \text{ ft/s}$.

- (a)



$$y^2 = (90-x)^2 + 90^2 \Rightarrow 2y \frac{dy}{dt} = 2(90-x) \left(-\frac{dx}{dt}\right). \text{ When } x = 45,$$

$$y = \sqrt{45^2 + 90^2} = 45\sqrt{5}, \text{ so } \frac{dy}{dt} = \frac{90-x}{y} \left(-\frac{dx}{dt}\right) = \frac{45}{45\sqrt{5}} (-24) = -\frac{24}{\sqrt{5}},$$

so the distance from second base is decreasing at a rate of $\frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s}$.

- (b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

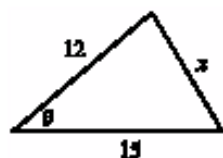
$$z^2 = x^2 + 90^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt}. \text{ When } x = 45, z = 45\sqrt{5}, \text{ so } \frac{dz}{dt} = \frac{45}{45\sqrt{5}} (24) = \frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s.}$$

28. We are given $d\theta/dt = 2^\circ/\text{min} = \frac{\pi}{90} \text{ rad/min}$. By the Law of Cosines,

$$x^2 = 12^2 + 15^2 - 2(12)(15) \cos \theta = 369 - 360 \cos \theta \Rightarrow$$

$$2x \frac{dx}{dt} = 360 \sin \theta \frac{d\theta}{dt} \Rightarrow \frac{dx}{dt} = \frac{180 \sin \theta}{x} \frac{d\theta}{dt}. \text{ When } \theta = 60^\circ,$$

$$x = \sqrt{369 - 360 \cos 60^\circ} = \sqrt{189} = 3\sqrt{21}, \text{ so } \frac{dx}{dt} = \frac{180 \sin 60^\circ}{3\sqrt{21}} \frac{\pi}{90} = \frac{\pi \sqrt{3}}{3\sqrt{21}} = \frac{\sqrt{7} \pi}{21} \approx 0.396 \text{ m/min.}$$



35. We are given that $\frac{dx}{dt} = 300$ km/h. By the Law of Cosines,

$$y^2 = x^2 + 1^2 - 2(1)(x) \cos 120^\circ = x^2 + 1 - 2x\left(-\frac{1}{2}\right) = x^2 + x + 1, \text{ so}$$

$$2y \frac{dy}{dt} = 2x \frac{dx}{dt} + \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{2x+1}{2y} \frac{dx}{dt}. \text{ After 1 minute, } x = \frac{300}{60} = 5 \text{ km} \Rightarrow$$

$$y = \sqrt{5^2 + 5 + 1} = \sqrt{31} \text{ km} \Rightarrow \frac{dy}{dt} = \frac{2(5)+1}{2\sqrt{31}}(300) = \frac{1650}{\sqrt{31}} \approx 296 \text{ km/h.}$$

