

14.  $f(x) = \frac{\ln x}{x}$ ,  $3 \leq x \leq 10$ .  $\Delta x = (10 - 3)/n = 7/n$  and  $x_i = 3 + i \Delta x = 3 + 7i/n$ .

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\ln(3 + 7i/n)}{3 + 7i/n} \cdot \frac{7}{n}.$$

16. (a)  $\Delta x = \frac{1-0}{n} = \frac{1}{n}$  and  $x_i = 0 + i \Delta x = \frac{i}{n}$ .  $A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n}.$

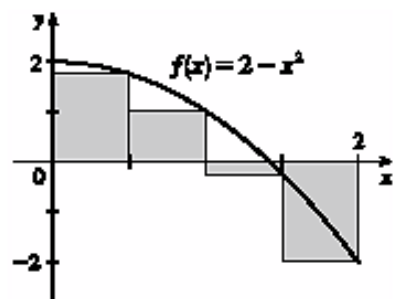
(b)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{n^3} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{1}{n^4} \sum_{i=1}^n i^3 = \lim_{n \rightarrow \infty} \frac{1}{n^4} \left[ \frac{n(n+1)}{2} \right]^2 = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{4n^2} = \frac{1}{4} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^2 = \frac{1}{4}$

1.  $R_4 = \sum_{i=1}^4 f(x_i) \Delta x$  [ $x_i^* = x_i$  is a right endpoint and  $\Delta x = 0.5$ ]

$$= 0.5 [f(0.5) + f(1) + f(1.5) + f(2)] \quad [f(x) = 2 - x^2]$$

$$= 0.5 [1.75 + 1 + (-0.25) + (-2)]$$

$$= 0.5(0.5) = 0.25$$



The Riemann sum represents the sum of the areas of the two rectangles above the  $x$ -axis minus the sum of the areas of the two rectangles below the  $x$ -axis; that is, the *net area* of the rectangles with respect to the  $x$ -axis.

12.  $\Delta x = (\pi - 0)/6 = \frac{\pi}{6}$ , so the endpoints are  $0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6},$  and  $\frac{6\pi}{6}$ , and the midpoints are  $\frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12},$  and  $\frac{11\pi}{12}$ . The Midpoint Rule gives

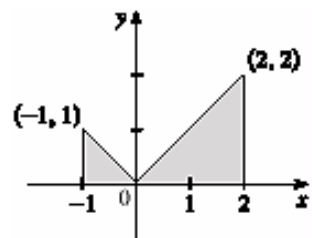
$$\int_0^\pi \sec(x/3) dx \approx \sum_{i=1}^6 f(\bar{x}_i) \Delta x = \frac{\pi}{6} (\sec \frac{\pi}{36} + \sec \frac{3\pi}{36} + \sec \frac{5\pi}{36} + \sec \frac{7\pi}{36} + \sec \frac{9\pi}{36} + \sec \frac{11\pi}{36}) \approx 3.9379.$$

25.  $f(x) = \frac{x}{1+x^5}$ ,  $a = 2$ ,  $b = 6$ , and  $\Delta x = \frac{6-2}{n} = \frac{4}{n}$ . Using Equation 3, we get  $x_i^* = x_i = 2 + i \Delta x = 2 + \frac{4i}{n}$ ,

$$\text{so } \int_2^6 \frac{x}{1+x^5} dx = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2 + \frac{4i}{n}}{1 + \left(2 + \frac{4i}{n}\right)^5} \cdot \frac{4}{n}.$$

35.  $\int_{-1}^2 |x| dx$  can be interpreted as the sum of the areas of the two shaded

triangles; that is,  $\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}.$



46. If  $0 \leq x \leq 2$ , then  $0 \leq x^3 \leq 8$ , so  $1 \leq x^3 + 1 \leq 9$  and  $1 \leq \sqrt{x^3 + 1} \leq 3$ .

Thus,  $1(2 - 0) \leq \int_0^2 \sqrt{x^3 + 1} \, dx \leq 3(2 - 0)$ ; that is,  $2 \leq \int_0^2 \sqrt{x^3 + 1} \, dx \leq 6$ .

47. If  $1 \leq x \leq 2$ , then  $\frac{1}{2} \leq \frac{1}{x} \leq 1$ , so  $\frac{1}{2}(2 - 1) \leq \int_1^2 \frac{1}{x} \, dx \leq 1(2 - 1)$  or  $\frac{1}{2} \leq \int_1^2 \frac{1}{x} \, dx \leq 1$ .