

4. Let  $u = x$ ,  $dv = e^{-x} dx \Rightarrow du = dx$ ,  $v = -e^{-x}$ . Then  $\int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C$ .

9. Let  $u = \ln(2x + 1)$ ,  $dv = dx \Rightarrow du = \frac{2}{2x + 1} dx$ ,  $v = x$ . Then

$$\begin{aligned}\int \ln(2x + 1) dx &= x \ln(2x + 1) - \int \frac{2x}{2x + 1} dx = x \ln(2x + 1) - \int \frac{(2x + 1) - 1}{2x + 1} dx \\ &= x \ln(2x + 1) - \int \left(1 - \frac{1}{2x + 1}\right) dx = x \ln(2x + 1) - x + \frac{1}{2} \ln(2x + 1) + C \\ &= \frac{1}{2}(2x + 1) \ln(2x + 1) - x + C\end{aligned}$$

19. Let  $u = y$ ,  $dv = \frac{dy}{e^{2y}} = e^{-2y} dy \Rightarrow du = dy$ ,  $v = -\frac{1}{2} e^{-2y}$ . Then

$$\int_0^1 \frac{y}{e^{2y}} dy = \left[-\frac{1}{2} y e^{-2y}\right]_0^1 + \frac{1}{2} \int_0^1 e^{-2y} dy = \left(-\frac{1}{2} e^{-2} + 0\right) - \frac{1}{4} [e^{-2y}]_0^1 = -\frac{1}{2} e^{-2} - \frac{1}{4} e^{-2} + \frac{1}{4} = \frac{1}{4} - \frac{3}{4} e^{-2}.$$

22. Let  $u = r^2$ ,  $dv = \frac{r}{\sqrt{4 + r^2}} dr \Rightarrow du = 2r dr$ ,  $v = \sqrt{4 + r^2}$ . By (6),

$$\begin{aligned}\int_0^1 \frac{r^3}{\sqrt{4 + r^2}} dr &= \left[r^2 \sqrt{4 + r^2}\right]_0^1 - 2 \int_0^1 r \sqrt{4 + r^2} dr = \sqrt{5} - \frac{2}{3} \left[(4 + r^2)^{3/2}\right]_0^1 \\ &= \sqrt{5} - \frac{2}{3}(5)^{3/2} + \frac{2}{3}(8) = \sqrt{5} \left(1 - \frac{10}{3}\right) + \frac{16}{3} = \frac{16}{3} - \frac{7}{3}\sqrt{5}\end{aligned}$$

42. Suppose  $f(0) = g(0) = 0$  and let  $u = f(x)$ ,  $dv = g''(x) dx \Rightarrow du = f'(x) dx$ ,  $v = g'(x)$ .

$$\text{Then } \int_0^a f(x) g''(x) dx = [f(x) g'(x)]_0^a - \int_0^a f'(x) g'(x) dx = f(a) g'(a) - \int_0^a f'(x) g'(x) dx.$$

Now let  $U = f'(x)$ ,  $dV = g'(x) dx \Rightarrow dU = f''(x) dx$  and  $V = g(x)$ , so

$$\int_0^a f'(x) g'(x) dx = [f'(x) g(x)]_0^a - \int_0^a f''(x) g(x) dx = f'(a) g(a) - \int_0^a f''(x) g(x) dx.$$

Combining the two results, we get  $\int_0^a f(x) g''(x) dx = f(a) g'(a) - f'(a) g(a) + \int_0^a f''(x) g(x) dx$ .

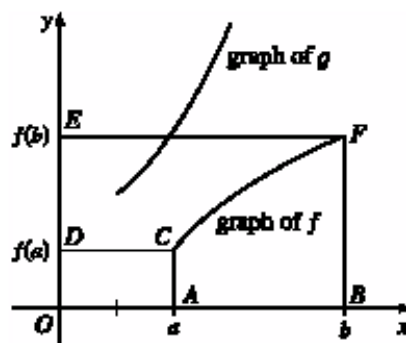
44. (a) Take  $g(x) = x$  and  $g'(x) = 1$  in Equation 1.

(b) By part (a),  $\int_a^b f(x) dx = bf(b) - af(a) - \int_a^b x f'(x) dx$ . Now let  $y = f(x)$ , so that  $x = g(y)$  and  $dy = f'(x) dx$ .

Then  $\int_a^b x f'(x) dx = \int_{f(a)}^{f(b)} g(y) dy$ . The result follows.

(c) Part (b) says that

$$\begin{aligned} \text{the area of region } ABFC &= bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y) dy \\ &= (\text{area of rectangle } OBF E) - (\text{area of rectangle } OACD) - (\text{area of region } DCFE) \end{aligned}$$



(d) We have  $f(x) = \ln x$ , so  $f^{-1}(x) = e^x$ , and since  $g = f^{-1}$ , we have  $g(y) = e^y$ . By part (b),

$$\int_1^e \ln x dx = e \ln e - 1 \ln 1 - \int_{\ln 1}^{\ln e} e^y dy = e - \int_0^1 e^y dy = e - [e^y]_0^1 = e - (e - 1) = 1.$$