

$$5. \int_0^{\pi/2} \cos^2 \theta \, d\theta = \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2\theta) \, d\theta \quad [\text{half-angle identity}] = \frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\pi/2} = \frac{1}{2} \left[\left(\frac{\pi}{2} + 0 \right) - (0 + 0) \right] = \frac{\pi}{4}$$

$$13. \int \cos^2 x \tan^3 x \, dx = \int \frac{\sin^3 x}{\cos x} \, dx \stackrel{c}{=} \int \frac{(1-u^2)(-du)}{u} = \int \left[\frac{-1}{u} + u \right] du$$

$$= -\ln|u| + \frac{1}{2}u^2 + C = \frac{1}{2}\cos^2 x - \ln|\cos x| + C$$

$$16. \int \cos^2 x \sin 2x \, dx = 2 \int \cos^3 x \sin x \, dx \stackrel{c}{=} -2 \int u^3 \, du = -\frac{1}{2}u^4 + C = -\frac{1}{2}\cos^4 x + C$$

$$25. \int \tan^3 x \sec x \, dx = \int \tan^2 x \sec x \tan x \, dx = \int (\sec^2 x - 1) \sec x \tan x \, dx$$

$$= \int (u^2 - 1) \, du \quad [u = \sec x, du = \sec x \tan x \, dx]$$

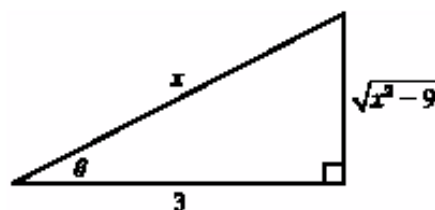
$$= \frac{1}{3}u^3 - u + C = \frac{1}{3}\sec^3 x - \sec x + C$$

$$37. \text{ Let } x = 3 \sec \theta, \text{ where } 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}.$$

Then $dx = 3 \sec \theta \tan \theta \, d\theta$ and

$$\sqrt{x^2 - 9} = \sqrt{9 \sec^2 \theta - 9} = \sqrt{9(\sec^2 \theta - 1)} = \sqrt{9 \tan^2 \theta}$$

$$= 3|\tan \theta| = 3 \tan \theta \quad \text{for the relevant values of } \theta.$$



$$\int \frac{1}{x^2 \sqrt{x^2 - 9}} \, dx = \int \frac{1}{9 \sec^2 \theta \cdot 3 \tan \theta} 3 \sec \theta \tan \theta \, d\theta = \frac{1}{9} \int \cos \theta \, d\theta = \frac{1}{9} \sin \theta + C = \frac{1}{9} \frac{\sqrt{x^2 - 9}}{x} + C$$

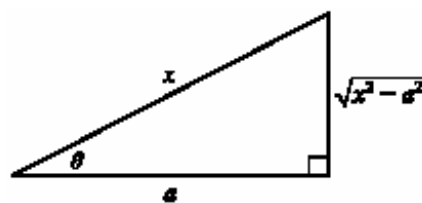
Note that $-\sec(\theta + \pi) = \sec \theta$, so the figure is sufficient for the case $\pi \leq \theta < \frac{3\pi}{2}$.

$$44. \text{ Let } x = a \sec \theta, \text{ where } 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}.$$

Then $dx = a \sec \theta \tan \theta \, d\theta$ and $\sqrt{x^2 - a^2} = a \tan \theta$, so

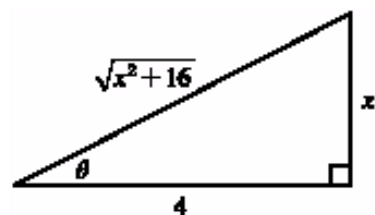
$$\int \frac{\sqrt{x^2 - a^2}}{x^4} \, dx = \int \frac{a \tan \theta}{a^4 \sec^4 \theta} a \sec \theta \tan \theta \, d\theta = \frac{1}{a^2} \int \sin^2 \theta \cos \theta \, d\theta$$

$$= \frac{1}{3a^2} \sin^3 \theta + C = \frac{(x^2 - a^2)^{3/2}}{3a^2 x^3} + C$$



45. Let $x = 4 \tan \theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then $dx = 4 \sec^2 \theta d\theta$ and

$$\begin{aligned}\sqrt{x^2 + 16} &= \sqrt{16 \tan^2 \theta + 16} = \sqrt{16(\tan^2 \theta + 1)} = \sqrt{16 \sec^2 \theta} \\ &= 4 |\sec \theta| = 4 \sec \theta \text{ for the relevant values of } \theta.\end{aligned}$$



$$\begin{aligned}\int \frac{dx}{\sqrt{x^2 + 16}} &= \int \frac{4 \sec^2 \theta d\theta}{4 \sec \theta} = \int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C_1 = \ln \left| \frac{\sqrt{x^2 + 16}}{4} + \frac{x}{4} \right| + C_1 \\ &= \ln |\sqrt{x^2 + 16} + x| - \ln |4| + C_1 = \ln(\sqrt{x^2 + 16} + x) + C, \text{ where } C = C_1 - \ln 4.\end{aligned}$$

(Since $\sqrt{x^2 + 16} + x > 0$, we don't need the absolute value.)

58. Let $u = \sin t$, $du = \cos t dt$. Then

$$\begin{aligned}\int_0^{\pi/2} \frac{\cos t}{\sqrt{1 + \sin^2 t}} dt &= \int_0^1 \frac{1}{\sqrt{1 + u^2}} du = \int_0^{\pi/4} \frac{1}{\sec \theta} \sec^2 \theta d\theta \quad \left[\begin{array}{l} \text{where } u = \tan \theta, du = \sec^2 \theta d\theta, \\ \text{and } \sqrt{1 + u^2} = \sec \theta \end{array} \right] \\ &= \int_0^{\pi/4} \sec \theta d\theta = \left[\ln |\sec \theta + \tan \theta| \right]_0^{\pi/4} \quad [\text{by (1)}] \\ &= \ln(\sqrt{2} + 1) - \ln(1 + 0) = \ln(\sqrt{2} + 1)\end{aligned}$$