

$$6. \frac{d}{dx}(y^5 + x^2 y^3) = \frac{d}{dx}(1 + x^4 y) \Rightarrow 5y^4 y' + x^2 \cdot 3y^2 y' + y^3 \cdot 2x = 0 + x^4 y' + y \cdot 4x^3 \Rightarrow$$

$$y'(5y^4 + 3x^2 y^2 - x^4) = 4x^3 y - 2xy^3 \Rightarrow y' = \frac{4x^3 y - 2xy^3}{5y^4 + 3x^2 y^2 - x^4}$$

$$8. \frac{d}{dx}(1 + x) = \frac{d}{dx}[\sin(xy^2)] \Rightarrow 1 = [\cos(xy^2)](x \cdot 2yy' + y^2 \cdot 1) \Rightarrow 1 = 2xy \cos(xy^2)y' + y^2 \cos(xy^2) \Rightarrow$$

$$1 - y^2 \cos(xy^2) = 2xy \cos(xy^2)y' \Rightarrow y' = \frac{1 - y^2 \cos(xy^2)}{2xy \cos(xy^2)}$$

$$18. x^2 + 2xy - y^2 + x = 2 \Rightarrow 2x + 2(xy' + y \cdot 1) - 2yy' + 1 = 0 \Rightarrow 2xy' - 2yy' = -2x - 2y - 1 \Rightarrow$$

$$y'(2x - 2y) = -2x - 2y - 1 \Rightarrow y' = \frac{-2x - 2y - 1}{2x - 2y}. \text{ When } x = 1 \text{ and } y = 2, \text{ we have } y' = \frac{-2 - 4 - 1}{2 - 4} = \frac{-7}{-2} = \frac{7}{2},$$

so an equation of the tangent line is  $y - 2 = \frac{7}{2}(x - 1)$  or  $y = \frac{7}{2}x - \frac{3}{2}$ .

34. The circles  $x^2 + y^2 = ax$  and  $x^2 + y^2 = by$  intersect at the origin where the tangents are vertical and horizontal.

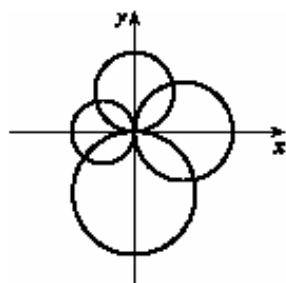
If  $(x_0, y_0)$  is the other point of intersection, then  $x_0^2 + y_0^2 = ax_0$  (1) and  $x_0^2 + y_0^2 = by_0$  (2).

$$\text{Now } x^2 + y^2 = ax \Rightarrow 2x + 2yy' = a \Rightarrow y' = \frac{a - 2x}{2y} \text{ and } x^2 + y^2 = by \Rightarrow$$

$$2x + 2yy' = by' \Rightarrow y' = \frac{2x}{b - 2y}. \text{ Thus, the curves are orthogonal at } (x_0, y_0) \Leftrightarrow$$

$$\frac{a - 2x_0}{2y_0} = -\frac{b - 2y_0}{2x_0} \Leftrightarrow 2ax_0 - 4x_0^2 = 4y_0^2 - 2by_0 \Leftrightarrow ax_0 + by_0 = 2(x_0^2 + y_0^2),$$

which is true by (1) and (2).



$$43. (a) y = J(x) \text{ and } xy'' + y' + xy = 0 \Rightarrow xJ''(x) + J'(x) + xJ(x) = 0. \text{ If } x = 0, \text{ we have } 0 + J'(0) + 0 = 0,$$

so  $J'(0) = 0$ .

$$(b) \text{ Differentiating } xy'' + y' + xy = 0 \text{ implicitly, we get } xy''' + y'' \cdot 1 + y'' + xy' + y \cdot 1 = 0 \Rightarrow$$

$$xy''' + 2y'' + xy' + y = 0, \text{ so } xJ'''(x) + 2J''(x) + xJ'(x) + J(x) = 0. \text{ If } x = 0, \text{ we have}$$

$$0 + 2J''(0) + 0 + 1 \quad [J(0) = 1 \text{ is given}] = 0 \Rightarrow 2J''(0) = -1 \Rightarrow J''(0) = -\frac{1}{2}.$$