

16.  $f(x) = x + \cos x \Rightarrow f'(x) = 1 - \sin x \geq 0$ , with equality only if  $x = \frac{\pi}{2} + 2n\pi$ . So  $f$  is increasing on  $\mathbb{R}$ , and hence, 1-1.

By inspection,  $f(0) = 0 + \cos 0 = 1$ , so  $f^{-1}(1) = 0$ .

20.  $m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{m_0^2}{m^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{m_0^2}{m^2} \Rightarrow v^2 = c^2 \left(1 - \frac{m_0^2}{m^2}\right) \Rightarrow v = c \sqrt{1 - \frac{m_0^2}{m^2}}$ .

This formula gives us the speed  $v$  of the particle in terms of its mass  $m$ , that is,  $v = f^{-1}(m)$ .

32. (a)  $x_1 \neq x_2 \Rightarrow x_1 - 2 \neq x_2 - 2 \Rightarrow \sqrt{x_1 - 2} \neq \sqrt{x_2 - 2} \Rightarrow f(x_1) \neq f(x_2)$ , so  $f$  is 1-1.

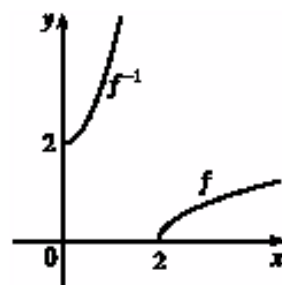
(b)  $f(6) = 2$ , so  $f^{-1}(2) = 6$ . Also  $f'(x) = \frac{1}{2\sqrt{x-2}}$ , so  $(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(6)} = \frac{1}{1/4} = 4$ .

(c)  $y = \sqrt{x-2} \Rightarrow y^2 = x-2 \Rightarrow x = y^2 + 2$ .

Interchange  $x$  and  $y$ :  $y = x^2 + 2$ . So  $f^{-1}(x) = x^2 + 2$ .

Domain =  $[0, \infty)$ , range =  $[2, \infty)$ .

(d)  $f^{-1}(x) = x^2 + 2 \Rightarrow (f^{-1})'(x) = 2x \Rightarrow (f^{-1})'(2) = 4$ .



36.  $f(1) = 2 \Rightarrow f^{-1}(2) = 1$ , and  $f(x) = x^5 - x^3 + 2x \Rightarrow f'(x) = 5x^4 - 3x^2 + 2$  and  $f'(1) = 4$ . Thus,

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{4}.$$

44. (a)  $\log_8 2 = \frac{1}{3}$  since  $8^{1/3} = 2$ .

(b)  $\ln e^{\sqrt{2}} = \sqrt{2}$

46. (a)  $2^{(\log_2 3 + \log_2 5)} = 2^{\log_2 15} = 15$  [Or:  $2^{(\log_2 3 + \log_2 5)} = 2^{\log_2 3} \cdot 2^{\log_2 5} = 3 \cdot 5 = 15$ ]

(b)  $e^{3 \ln 2} = e^{\ln(2^3)} = e^{\ln 8} = 8$  [Or:  $e^{3 \ln 2} = (e^{\ln 2})^3 = 2^3 = 8$ ]

50.  $\ln \frac{3x^2}{(x+1)^5} = \ln 3x^2 - \ln(x+1)^5 = \ln 3 + \ln x^2 - 5 \ln(x+1) = \ln 3 + 2 \ln x - 5 \ln(x+1)$

62. (a)  $e^{2x+3} - 7 = 0 \Rightarrow e^{2x+3} = 7 \Rightarrow 2x+3 = \ln 7 \Rightarrow 2x = \ln 7 - 3 \Rightarrow x = \frac{1}{2}(\ln 7 - 3)$

(b)  $\ln(5-2x) = -3 \Rightarrow 5-2x = e^{-3} \Rightarrow 2x = 5 - e^{-3} \Rightarrow x = \frac{1}{2}(5 - e^{-3})$

66. (a)  $2 < \ln x < 9 \Rightarrow e^2 < e^{\ln x} < e^9 \Rightarrow e^2 < x < e^9 \Rightarrow x \in (e^2, e^9)$

(b)  $e^{2-3x} > 4 \Rightarrow \ln e^{2-3x} > \ln 4 \Rightarrow 2-3x > \ln 4 \Rightarrow -3x > \ln 4 - 2 \Rightarrow x < -\frac{1}{3}(\ln 4 - 2) \Rightarrow x \in (-\infty, \frac{1}{3}(2 - \ln 4))$