

2. (a) $f(x) = x^4 - 4x - 1 \Rightarrow f'(x) = 4x^3 - 4 = 4(x^3 - 1) = 4(x - 1)(x^2 + x + 1)$. So $f'(x) > 0 \Leftrightarrow x - 1 > 0$ [$4(x^2 + x + 1) > 0$] $\Leftrightarrow x > 1$. Thus, f is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.
- (b) f changes from decreasing to increasing at its only critical number, $x = 1$. Thus, $f(1) = -4$ is a local minimum value.
- (c) $f'(x) = 4x^3 - 4 \Rightarrow f''(x) = 12x^2$. $f''(x) > 0$ for all x except $x = 0$. Thus, f is CU on $(-\infty, 0)$ and $(0, \infty)$.

Moreover, since f' is increasing on $(-\infty, \infty)$, f is CU on $(-\infty, \infty)$. There are no IPs.

$$10. f(x) = \frac{x}{x^2 + 4} \Rightarrow f'(x) = \frac{(x^2 + 4) \cdot 1 - x(2x)}{(x^2 + 4)^2} = \frac{4 - x^2}{(x^2 + 4)^2} = \frac{(2 + x)(2 - x)}{(x^2 + 4)^2}.$$

First Derivative Test: $f'(x) > 0 \Rightarrow -2 < x < 2$ and $f'(x) < 0 \Rightarrow x > 2$ or $x < -2$. Since f' changes from positive to negative at $x = 2$, $f(2) = \frac{1}{4}$ is a local maximum value; and since f' changes from negative to positive at $x = -2$, $f(-2) = -\frac{1}{4}$ is a local minimum value.

Second Derivative Test:

$$f''(x) = \frac{(x^2 + 4)^2(-2x) - (4 - x^2) \cdot 2(x^2 + 4)(2x)}{[(x^2 + 4)^2]^2} = \frac{-2x(x^2 + 4)[(x^2 + 4) + 2(4 - x^2)]}{(x^2 + 4)^4} = \frac{-2x(12 - x^2)}{(x^2 + 4)^3}$$

$f'(x) = 0 \Leftrightarrow x = \pm 2$. $f''(-2) = \frac{1}{16} > 0 \Rightarrow f(-2) = -\frac{1}{4}$ is a local minimum value.

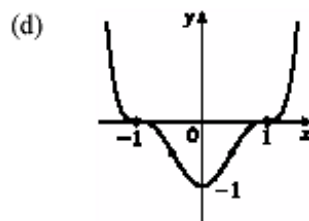
$f''(2) = -\frac{1}{16} < 0 \Rightarrow f(2) = \frac{1}{4}$ is a local maximum value.

Preference: Since calculating the second derivative is fairly difficult, the First Derivative Test is easier to use for this function.

28. (a) $h(x) = (x^2 - 1)^3 \Rightarrow h'(x) = 6x(x^2 - 1)^2 \geq 0 \Leftrightarrow x > 0$ ($x \neq 1$), so h is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.
- (b) $h(0) = -1$ is a local minimum value.
- (c) $h''(x) = 6(x^2 - 1)^2 + 24x^2(x^2 - 1) = 6(x^2 - 1)(5x^2 - 1)$. The roots ± 1 and $\pm \frac{1}{\sqrt{5}}$ divide \mathbb{R} into five intervals.

Interval	$x^2 - 1$	$5x^2 - 1$	$h''(x)$	Concavity
$x < -1$	+	+	+	upward
$-1 < x < -\frac{1}{\sqrt{5}}$	-	+	-	downward
$-\frac{1}{\sqrt{5}} < x < \frac{1}{\sqrt{5}}$	-	-	+	upward
$\frac{1}{\sqrt{5}} < x < 1$	-	+	-	downward
$x > 1$	+	+	+	upward

From the table, we see that h is CU on $(-\infty, -1)$, $(-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}})$ and $(1, \infty)$, and CD on $(-1, -\frac{1}{\sqrt{5}})$ and $(\frac{1}{\sqrt{5}}, 1)$. There are IPs at $(\pm 1, 0)$ and $(\pm \frac{1}{\sqrt{5}}, -\frac{64}{125})$.

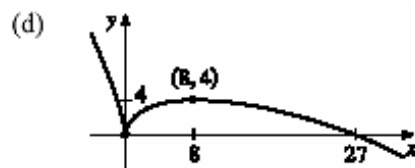


30. (a) $B(x) = 3x^{2/3} - x \Rightarrow B'(x) = 2x^{-1/3} - 1 = \frac{2}{\sqrt[3]{x}} - 1 = \frac{2 - \sqrt[3]{x}}{\sqrt[3]{x}}$. $B'(x) > 0$ if $0 < x < 8$ and $B'(x) < 0$ if $x < 0$ or $x > 8$, so B is decreasing on $(-\infty, 0)$ and $(8, \infty)$, and B is increasing on $(0, 8)$.

(b) $B(0) = 0$ is a local minimum value. $B(8) = 4$ is a local maximum value.

(c) $B''(x) = -\frac{2}{3}x^{-4/3} = \frac{-2}{3x^{4/3}}$, so $B''(x) < 0$ for all $x \neq 0$.

B is CD on $(-\infty, 0)$ and $(0, \infty)$. No IP



38. (a) $\lim_{x \rightarrow \pi/2^-} x \tan x = \infty$ and $\lim_{x \rightarrow -\pi/2^+} x \tan x = \infty$, so $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$ are VAs.

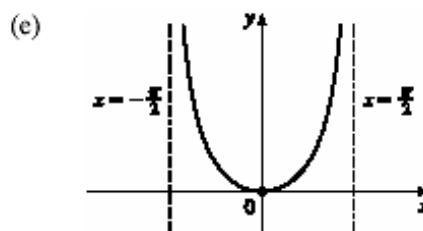
(b) $f(x) = x \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. $f'(x) = x \sec^2 x + \tan x > 0 \Leftrightarrow$

$0 < x < \frac{\pi}{2}$, so f increases on $(0, \frac{\pi}{2})$ and decreases on $(-\frac{\pi}{2}, 0)$.

(c) $f(0) = 0$ is a local minimum value.

(d) $f''(x) = 2 \sec^2 x + 2x \tan x \sec^2 x > 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$,

so f is CU on $(-\frac{\pi}{2}, \frac{\pi}{2})$. No IP



52. (a) Let $f(x) = e^x - 1 - x$. Now $f(0) = e^0 - 1 - 0 = 0$, and for $x \geq 0$, we have $f'(x) = e^x - 1 \geq 0$. Now, since $f(0) = 0$ and f is increasing on $[0, \infty)$, $f(x) \geq 0$ for $x \geq 0 \Rightarrow e^x - 1 - x \geq 0 \Rightarrow e^x \geq 1 + x$.

(b) Let $f(x) = e^x - 1 - x - \frac{1}{2}x^2$. Thus, $f'(x) = e^x - 1 - x$, which is positive for $x \geq 0$ by part (a). Thus, $f(x)$ is increasing on $(0, \infty)$, so on that interval, $0 = f(0) \leq f(x) = e^x - 1 - x - \frac{1}{2}x^2 \Rightarrow e^x \geq 1 + x + \frac{1}{2}x^2$.

(c) By part (a), the result holds for $n = 1$. Suppose that $e^x \geq 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^k}{k!}$ for $x \geq 0$.

Let $f(x) = e^x - 1 - x - \frac{x^2}{2!} - \cdots - \frac{x^k}{k!} - \frac{x^{k+1}}{(k+1)!}$. Then $f'(x) = e^x - 1 - x - \frac{x^2}{2!} - \cdots - \frac{x^k}{k!} \geq 0$ by assumption. Hence,

$f(x)$ is increasing on $(0, \infty)$. So $0 \leq x$ implies that $0 = f(0) \leq f(x) = e^x - 1 - x - \frac{x^2}{2!} - \cdots - \frac{x^k}{k!} - \frac{x^{k+1}}{(k+1)!}$, and hence

$e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \frac{x^{k+1}}{(k+1)!}$ for $x \geq 0$. Therefore, for $x \geq 0$, $e^x \geq 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$ for every positive

integer n , by mathematical induction.