

$$2. (a) \frac{x-1}{x^3+x^2} = \frac{x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$(b) \frac{x-1}{x^3+x} = \frac{x-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

15. $\frac{2x+3}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \Rightarrow 2x+3 = A(x+1) + B$. Take $x = -1$ to get $B = 1$, and equate coefficients of x to get $A = 2$. Now

$$\begin{aligned} \int_0^1 \frac{2x+3}{(x+1)^2} dx &= \int_0^1 \left[\frac{2}{x+1} + \frac{1}{(x+1)^2} \right] dx = \left[2\ln(x+1) - \frac{1}{x+1} \right]_0^1 \\ &= 2\ln 2 - \frac{1}{2} - (2\ln 1 - 1) = 2\ln 2 + \frac{1}{2} \end{aligned}$$

$$22. \frac{x^2-x+6}{x^3+3x} = \frac{x^2-x+6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}. \text{ Multiply by } x(x^2+3) \text{ to get } x^2-x+6 = A(x^2+3) + (Bx+C)x.$$

Substituting 0 for x gives $6 = 3A \Leftrightarrow A = 2$. The coefficients of the x^2 -terms must be equal, so $1 = A + B \Rightarrow B = 1 - 2 = -1$. The coefficients of the x -terms must be equal, so $-1 = C$. Thus,

$$\begin{aligned} \int \frac{x^2-x+6}{x^3+3x} dx &= \int \left(\frac{2}{x} + \frac{-x-1}{x^2+3} \right) dx = \int \left(\frac{2}{x} - \frac{x}{x^2+3} - \frac{1}{x^2+3} \right) dx \\ &= 2\ln|x| - \frac{1}{2}\ln(x^2+3) - \frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + C \end{aligned}$$

$$30. \frac{x^3}{x^3+1} = \frac{(x^3+1)-1}{x^3+1} = 1 - \frac{1}{x^3+1} = 1 - \left(\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \right) \Rightarrow 1 = A(x^2-x+1) + (Bx+C)(x+1).$$

Equate the terms of degree 2, 1 and 0 to get $0 = A + B$, $0 = -A + B + C$, $1 = A + C$. Solve the three equations to get

$A = \frac{1}{3}$, $B = -\frac{1}{3}$, and $C = \frac{2}{3}$. So

$$\begin{aligned} \int \frac{x^3}{x^3+1} dx &= \int \left[1 - \frac{\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x - \frac{2}{3}}{x^2-x+1} \right] dx = x - \frac{1}{3}\ln|x+1| + \frac{1}{6} \int \frac{2x-1}{x^2-x+1} dx - \frac{1}{2} \int \frac{dx}{(x-\frac{1}{2})^2 + \frac{3}{4}} \\ &= x - \frac{1}{3}\ln|x+1| + \frac{1}{6}\ln(x^2-x+1) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}(2x-1)\right) + K \end{aligned}$$

$$36. \text{ Let } u = \sqrt[3]{x}. \text{ Then } x = u^3, dx = 3u^2 du \Rightarrow$$

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}} dx = \int_0^1 \frac{3u^2 du}{1+u} = \int_0^1 \left(3u - 3 + \frac{3}{1+u} \right) du = \left[\frac{3}{2}u^2 - 3u + 3\ln(1+u) \right]_0^1 = 3\left(\ln 2 - \frac{1}{2}\right).$$

42. Let $u = \tan^{-1} x$, $dv = x \, dx \Rightarrow du = dx/(1+x^2)$, $v = \frac{1}{2}x^2$.

Then $\int x \tan^{-1} x \, dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx$. To evaluate the last integral, use long division or

observe that $\int \frac{x^2}{1+x^2} \, dx = \int \frac{(1+x^2)-1}{1+x^2} \, dx = \int 1 \, dx - \int \frac{1}{1+x^2} \, dx = x - \tan^{-1} x + C_1$.

So $\int x \tan^{-1} x \, dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}(x - \tan^{-1} x + C_1) = \frac{1}{2}(x^2 \tan^{-1} x + \tan^{-1} x - x) + C$.