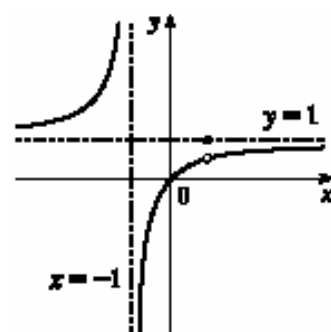


$$16. f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{x}{x+1} = \frac{1}{2},$$

but  $f(1) = 1$ , so  $f$  is discontinuous at 1.



22. The sine and cosine functions are continuous everywhere by Theorem 6, so  $F(x) = \sin(\cos(\sin x))$ , which is the composite of sine, cosine, and (once again) sine, is continuous everywhere by Theorem 8.

$$29. f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } x > 1 \end{cases}$$

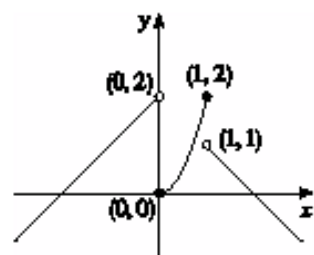
$f$  is continuous on  $(-\infty, 0)$ ,  $(0, 1)$ , and  $(1, \infty)$  since on each of

these intervals it is a polynomial. Now  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (x + 2) = 2$  and

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2x^2 = 0$ , so  $f$  is discontinuous at 0. Since  $f(0) = 0$ ,  $f$  is continuous from the right at 0. Also

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 2x^2 = 2$  and  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2 - x) = 1$ , so  $f$  is discontinuous at 1. Since  $f(1) = 2$ ,

$f$  is continuous from the left at 1.



33. (a)  $f(x) = \frac{x^2 - 2x - 8}{x + 2} = \frac{(x - 4)(x + 2)}{x + 2}$  has a removable discontinuity at  $-2$  because  $g(x) = x - 4$  is continuous on  $\mathbb{R}$  and  $f(x) = g(x)$  for  $x \neq -2$ . [The discontinuity is removed by defining  $f(-2) = -6$ .]

- (b)  $f(x) = \frac{x - 7}{|x - 7|} \Rightarrow \lim_{x \rightarrow 7^-} f(x) = -1$  and  $\lim_{x \rightarrow 7^+} f(x) = 1$ . Thus,  $\lim_{x \rightarrow 7} f(x)$  does not exist, so the discontinuity is not removable. (It is a jump discontinuity.)

- (c)  $f(x) = \frac{x^3 + 64}{x + 4} = \frac{(x + 4)(x^2 - 4x + 16)}{x + 4}$  has a removable discontinuity at  $-4$  because  $g(x) = x^2 - 4x + 16$  is continuous on  $\mathbb{R}$  and  $f(x) = g(x)$  for  $x \neq -4$ . [The discontinuity is removed by defining  $f(-4) = 48$ .]

- (d)  $f(x) = \frac{3 - \sqrt{x}}{9 - x} = \frac{3 - \sqrt{x}}{(3 - \sqrt{x})(3 + \sqrt{x})}$  has a removable discontinuity at 9 because  $g(x) = \frac{1}{3 + \sqrt{x}}$  is continuous on  $[0, \infty)$  and  $f(x) = g(x)$  for  $x \neq 9$ . [The discontinuity is removed by defining  $f(9) = \frac{1}{6}$ .]

37.  $f(x) = x^4 + x - 3$  is continuous on the interval  $[1, 2]$ ,  $f(1) = -1$ , and  $f(2) = 15$ . Since  $-1 < 0 < 15$ , there is a number  $c$  in  $(1, 2)$  such that  $f(c) = 0$  by the Intermediate Value Theorem. Thus, there is a root of the equation  $x^4 + x - 3 = 0$  in the interval  $(1, 2)$ .

39.  $f(x) = \cos x - x$  is continuous on the interval  $[0, 1]$ ,  $f(0) = 1$ , and  $f(1) = \cos 1 - 1 \approx -0.46$ . Since  $-0.46 < 0 < 1$ , there is a number  $c$  in  $(0, 1)$  such that  $f(c) = 0$  by the Intermediate Value Theorem. Thus, there is a root of the equation  $\cos x - x = 0$ , or  $\cos x = x$ , in the interval  $(0, 1)$ .