

2. (a)  $\lim_{x \rightarrow \infty} g(x) = 2$       (b)  $\lim_{x \rightarrow -\infty} g(x) = -2$       (c)  $\lim_{x \rightarrow 3} g(x) = \infty$   
 (d)  $\lim_{x \rightarrow 0} g(x) = -\infty$       (e)  $\lim_{x \rightarrow -2^+} g(x) = -\infty$       (f) Vertical:  $x = -2, x = 0, x = 3$ ; Horizontal:  $y = -2, y = 2$

$$22. \lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{9x^2+1}} = \lim_{x \rightarrow \infty} \frac{(x+2)/x}{\sqrt{9x^2+1}/\sqrt{x^2}} = \lim_{x \rightarrow \infty} \frac{1+2/x}{\sqrt{9+1/x^2}} = \frac{1+0}{\sqrt{9+0}} = \frac{1}{3}$$

25.  $\lim_{x \rightarrow \infty} \cos x$  does not exist because as  $x$  increases  $\cos x$  does not approach any one value, but oscillates between 1 and  $-1$ .

$$30. \lim_{x \rightarrow \infty} (x^2 - x^4) = \lim_{x \rightarrow \infty} x^2(1 - x^2) = -\infty \text{ since } x^2 \rightarrow \infty \text{ and } 1 - x^2 \rightarrow -\infty.$$

$$43. \lim_{x \rightarrow \infty} \frac{4x-1}{x} = \lim_{x \rightarrow \infty} \left(4 - \frac{1}{x}\right) = 4 \text{ and } \lim_{x \rightarrow \infty} \frac{4x^2+3x}{x^2} = \lim_{x \rightarrow \infty} \left(4 + \frac{3}{x}\right) = 4. \text{ Therefore, by the Squeeze Theorem, } \lim_{x \rightarrow \infty} f(x) = 4.$$

55. Suppose that  $\lim_{x \rightarrow \infty} f(x) = L$ . Then for every  $\varepsilon > 0$  there is a corresponding positive number  $N$  such that  $|f(x) - L| < \varepsilon$  whenever  $x > N$ . If  $t = 1/x$ , then  $x > N \Leftrightarrow 0 < 1/x < 1/N \Leftrightarrow 0 < t < 1/N$ . Thus, for every  $\varepsilon > 0$  there is a corresponding  $\delta > 0$  (namely  $1/N$ ) such that  $|f(1/t) - L| < \varepsilon$  whenever  $0 < t < \delta$ . This proves that

$$\lim_{t \rightarrow 0^+} f(1/t) = L = \lim_{x \rightarrow \infty} f(x).$$

Now suppose that  $\lim_{x \rightarrow -\infty} f(x) = L$ . Then for every  $\varepsilon > 0$  there is a corresponding negative number  $N$  such that

$|f(x) - L| < \varepsilon$  whenever  $x < N$ . If  $t = 1/x$ , then  $x < N \Leftrightarrow 1/N < 1/x < 0 \Leftrightarrow 1/N < t < 0$ . Thus, for every  $\varepsilon > 0$  there is a corresponding  $\delta > 0$  (namely  $-1/N$ ) such that  $|f(1/t) - L| < \varepsilon$  whenever  $-\delta < t < 0$ . This proves that

$$\lim_{t \rightarrow 0^-} f(1/t) = L = \lim_{x \rightarrow -\infty} f(x).$$