

$$\begin{aligned}
 28. f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{3(a+h)+1} - \sqrt{3a+1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(\sqrt{3a+3h+1} - \sqrt{3a+1})(\sqrt{3a+3h+1} + \sqrt{3a+1})}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} = \lim_{h \rightarrow 0} \frac{(3a+3h+1) - (3a+1)}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} \\
 &= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} = \lim_{h \rightarrow 0} \frac{3}{\sqrt{3a+3h+1} + \sqrt{3a+1}} = \frac{3}{2\sqrt{3a+1}}
 \end{aligned}$$

Note that the answers to Exercises 29–34 are not unique.

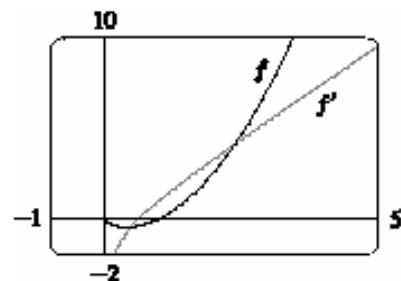
$$32. \text{ By Equation 5, } \lim_{x \rightarrow \pi/4} \frac{\tan x - 1}{x - \pi/4} = f'(\pi/4), \text{ where } f(x) = \tan x \text{ and } a = \pi/4.$$

$$\begin{aligned}
 23. G'(t) &= \lim_{h \rightarrow 0} \frac{G(t+h) - G(t)}{h} = \lim_{h \rightarrow 0} \frac{\frac{4(t+h)}{(t+h)+1} - \frac{4t}{t+1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{4(t+h)(t+1) - 4t(t+h+1)}{(t+h+1)(t+1)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(4t^2 + 4ht + 4t + 4h) - (4t^2 + 4ht + 4t)}{h(t+h+1)(t+1)} = \lim_{h \rightarrow 0} \frac{4h}{h(t+h+1)(t+1)} \\
 &= \lim_{h \rightarrow 0} \frac{4}{(t+h+1)(t+1)} = \frac{4}{(t+1)^2}
 \end{aligned}$$

Domain of G = domain of $G' = (-\infty, -1) \cup (-1, \infty)$.

$$\begin{aligned}
 26. (a) f'(t) &= \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} = \lim_{h \rightarrow 0} \frac{[(t+h)^2 - \sqrt{t+h}] - (t^2 - \sqrt{t})}{h} \\
 &= \lim_{h \rightarrow 0} \frac{t^2 + 2ht + h^2 - \sqrt{t+h} - t^2 + \sqrt{t}}{h} = \lim_{h \rightarrow 0} \left(\frac{2ht + h^2}{h} + \frac{\sqrt{t} - \sqrt{t+h}}{h} \right) \\
 &= \lim_{h \rightarrow 0} \left(\frac{h(2t+h)}{h} + \frac{\sqrt{t} - \sqrt{t+h}}{h} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \right) \\
 &= \lim_{h \rightarrow 0} \left(2t + h + \frac{t - (t+h)}{h(\sqrt{t} + \sqrt{t+h})} \right) = \lim_{h \rightarrow 0} \left(2t + h + \frac{-h}{h(\sqrt{t} + \sqrt{t+h})} \right) \\
 &= \lim_{h \rightarrow 0} \left(2t + h + \frac{-1}{\sqrt{t} + \sqrt{t+h}} \right) = 2t - \frac{1}{2\sqrt{t}}
 \end{aligned}$$

(b) Notice that $f'(t) = 0$ when f has a horizontal tangent, $f'(t)$ is positive when the tangents have positive slope, and $f'(t)$ is negative when the tangents have negative slope.



40. (a) $g'(0) = \lim_{x \rightarrow 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{x^{2/3} - 0}{x} = \lim_{x \rightarrow 0} \frac{1}{x^{1/3}}$, which does not exist.

(b) $g'(a) = \lim_{x \rightarrow a} \frac{g(x) - g(a)}{x - a} = \lim_{x \rightarrow a} \frac{x^{2/3} - a^{2/3}}{x - a} = \lim_{x \rightarrow a} \frac{(x^{1/3} - a^{1/3})(x^{1/3} + a^{1/3})}{(x^{1/3} - a^{1/3})(x^{2/3} + x^{1/3}a^{1/3} + a^{2/3})}$
 $= \lim_{x \rightarrow a} \frac{x^{1/3} + a^{1/3}}{x^{2/3} + x^{1/3}a^{1/3} + a^{2/3}} = \frac{2a^{1/3}}{3a^{2/3}} = \frac{2}{3a^{1/3}} \text{ or } \frac{2}{3}a^{-1/3}$

(c) $g(x) = x^{2/3}$ is continuous at $x = 0$ and

$$\lim_{x \rightarrow 0} |g'(x)| = \lim_{x \rightarrow 0} \frac{2}{3|x|^{1/3}} = \infty. \text{ This shows that}$$

g has a vertical tangent line at $x = 0$.

(d)

