

1. (a)  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$  since  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$  and  $\frac{\pi}{3}$  is in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

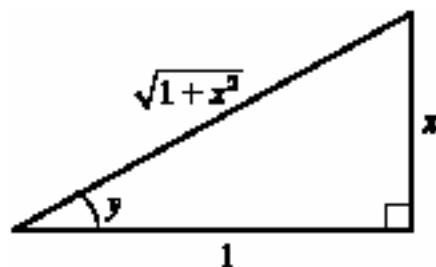
(b)  $\cos^{-1}(-1) = \pi$  since  $\cos \pi = -1$  and  $\pi$  is in  $[0, \pi]$ .

2. (a)  $\arctan(-1) = -\frac{\pi}{4}$  since  $\tan(-\frac{\pi}{4}) = -1$  and  $-\frac{\pi}{4}$  is in  $(-\frac{\pi}{2}, \frac{\pi}{2})$ .

(b)  $\csc^{-1} 2 = \frac{\pi}{6}$  since  $\csc \frac{\pi}{6} = 2$  and  $\frac{\pi}{6}$  is in  $(0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$ .

9. Let  $y = \tan^{-1} x$ . Then  $\tan y = x$ , so from the triangle

we see that  $\sin(\tan^{-1} x) = \sin y = \frac{x}{\sqrt{1+x^2}}$ .



16.  $y = \sqrt{\tan^{-1} x} = (\tan^{-1} x)^{1/2} \Rightarrow$

$$y' = \frac{1}{2}(\tan^{-1} x)^{-1/2} \cdot \frac{d}{dx}(\tan^{-1} x) = \frac{1}{2\sqrt{\tan^{-1} x}} \cdot \frac{1}{1+x^2} = \frac{1}{2\sqrt{\tan^{-1} x}(1+x^2)}$$

24.  $y = x \cos^{-1} x - \sqrt{1-x^2} \Rightarrow y' = \cos^{-1} x - \frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} = \cos^{-1} x$

25.  $y = \arctan(\cos \theta) \Rightarrow y' = \frac{1}{1+(\cos \theta)^2} (-\sin \theta) = -\frac{\sin \theta}{1+\cos^2 \theta}$

28.  $y = \tan^{-1}\left(\frac{x}{a}\right) + \ln \sqrt{\frac{x-a}{x+a}} = \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{2} \ln(x-a) - \frac{1}{2} \ln(x+a) \Rightarrow$

$$y' = \frac{a}{x^2+a^2} + \frac{1/2}{x-a} - \frac{1/2}{x+a} = \frac{a}{x^2+a^2} + \frac{a}{x^2-a^2} = \frac{2ax^2}{x^4-a^4}$$

37. Let  $t = e^x$ . As  $x \rightarrow \infty$ ,  $t \rightarrow \infty$ .  $\lim_{x \rightarrow \infty} \arctan(e^x) = \lim_{t \rightarrow \infty} \arctan t = \frac{\pi}{2}$  by (8).