

$$5. f(x) = x^2 \sin x, \Delta x = \frac{b-a}{n} = \frac{\pi-0}{8} = \frac{\pi}{8}$$

$$(a) M_8 = \frac{\pi}{8} \left[f\left(\frac{\pi}{16}\right) + f\left(\frac{3\pi}{16}\right) + f\left(\frac{5\pi}{16}\right) + \cdots + f\left(\frac{15\pi}{16}\right) \right] \approx 5.932957$$

$$(b) S_8 = \frac{\pi}{8 \cdot 3} \left[f(0) + 4f\left(\frac{\pi}{8}\right) + 2f\left(\frac{2\pi}{8}\right) + 4f\left(\frac{3\pi}{8}\right) + 2f\left(\frac{4\pi}{8}\right) + 4f\left(\frac{5\pi}{8}\right) + 2f\left(\frac{6\pi}{8}\right) + 4f\left(\frac{7\pi}{8}\right) + f(\pi) \right] \\ \approx 5.869247$$

$$\text{Actual: } \int_0^\pi x^2 \sin x \, dx \stackrel{84}{=} \left[-x^2 \cos x \right]_0^\pi + 2 \int_0^\pi x \cos x \, dx \stackrel{83}{=} \left[-\pi^2(-1) - 0 \right] + 2[\cos x + x \sin x]_0^\pi \\ = \pi^2 + 2[(-1+0) - (1+0)] = \pi^2 - 4 \approx 5.869604$$

$$\text{Errors: } E_M = \text{actual} - M_8 = \int_0^\pi x^2 \sin x \, dx - M_8 \approx -0.063353$$

$$E_S = \text{actual} - S_8 = \int_0^\pi x^2 \sin x \, dx - S_8 \approx 0.000357$$

$$16. f(z) = \sqrt{z}e^{-z}, \Delta z = \frac{1-0}{10} = \frac{1}{10}$$

$$(a) T_{10} = \frac{1}{10 \cdot 2} \{ f(0) + 2[f(0.1) + f(0.2) + \cdots + f(0.9)] + f(1) \} \approx 0.372299$$

$$(b) M_{10} = \frac{1}{10} [f(0.05) + f(0.15) + f(0.25) + \cdots + f(0.95)] \approx 0.380894$$

$$(c) S_{10} = \frac{1}{10 \cdot 3} [f(0) + 4f(0.1) + 2f(0.2) + 4f(0.3) + 2f(0.4) + 4f(0.5) + 2f(0.6) \\ + 4f(0.7) + 2f(0.8) + 4f(0.9) + f(1)] \\ \approx 0.376330$$

$$20. \text{ From Example 7(b), we take } K = 76e \text{ to get } |E_S| \leq 76e(1)^5/(180n^4) \leq 0.00001 \Rightarrow n^4 \geq 76e/[180(0.00001)] \Rightarrow \\ n \geq 18.4. \text{ Take } n = 20 \text{ (since } n \text{ must be even).}$$

40. $T_n = \frac{\Delta x}{2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$ and $M_n = \Delta x \sum_{i=1}^n f\left(x_i - \frac{\Delta x}{2}\right)$, so

$$\frac{1}{3}T_n + \frac{2}{3}M_n = \frac{1}{3}(T_n + 2M_n) = \frac{\Delta x}{3 \cdot 2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) + 4 \sum_{i=1}^n f\left(x_i - \frac{\Delta x}{2}\right) \right]$$

where $\Delta x = \frac{b-a}{n}$. Let $\delta x = \frac{b-a}{2n}$. Then $\Delta x = 2\delta x$, so

$$\begin{aligned} \frac{1}{3}T_n + \frac{2}{3}M_n &= \frac{\delta x}{3} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) + 4 \sum_{i=1}^n f(x_i - \delta x) \right] \\ &= \frac{1}{3}\delta x [f(x_0) + 4f(x_1 - \delta x) + 2f(x_1) + 4f(x_2 - \delta x) \\ &\quad + 2f(x_2) + \cdots + 2f(x_{n-1}) + 4f(x_n - \delta x) + f(x_n)] \end{aligned}$$

Since $x_0, x_1 - \delta x, x_1, x_2 - \delta x, x_2, \dots, x_{n-1}, x_n - \delta x, x_n$ are the subinterval endpoints for S_{2n} , and since $\delta x = \frac{b-a}{2n}$

is the width of the subintervals for S_{2n} , the last expression for $\frac{1}{3}T_n + \frac{2}{3}M_n$ is the usual expression for S_{2n} .

Therefore, $\frac{1}{3}T_n + \frac{2}{3}M_n = S_{2n}$.