

3. (a)  $f(x)$  approaches 2 as  $x$  approaches 1 from the left, so  $\lim_{x \rightarrow 1^-} f(x) = 2$ .  
 (b)  $f(x)$  approaches 3 as  $x$  approaches 1 from the right, so  $\lim_{x \rightarrow 1^+} f(x) = 3$ .  
 (c)  $\lim_{x \rightarrow 1} f(x)$  does not exist because the limits in part (a) and part (b) are not equal.  
 (d)  $f(x)$  approaches 4 as  $x$  approaches 5 from the left and from the right, so  $\lim_{x \rightarrow 5} f(x) = 4$ .  
 (e)  $f(5)$  is not defined, so it doesn't exist.
4. (a)  $\lim_{x \rightarrow 0} f(x) = 3$  (b)  $\lim_{x \rightarrow 3^-} f(x) = 4$  (c)  $\lim_{x \rightarrow 3^+} f(x) = 2$   
 (d)  $\lim_{x \rightarrow 3} f(x)$  does not exist because the limits in part (b) and part (c) are not equal. (e)  $f(3) = 3$

$$\begin{aligned} 21. \lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7} &= \lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7} \cdot \frac{\sqrt{x+2}+3}{\sqrt{x+2}+3} = \lim_{x \rightarrow 7} \frac{(x+2)-9}{(x-7)(\sqrt{x+2}+3)} \\ &= \lim_{x \rightarrow 7} \frac{x-7}{(x-7)(\sqrt{x+2}+3)} = \lim_{x \rightarrow 7} \frac{1}{\sqrt{x+2}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6} \end{aligned}$$

$$23. \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{\frac{4}{4+x}} = \lim_{x \rightarrow -4} \frac{\frac{x+4}{4x}}{\frac{4}{4+x}} = \lim_{x \rightarrow -4} \frac{x+4}{4x(4+x)} = \lim_{x \rightarrow -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$$

31.  $-1 \leq \cos(2/x) \leq 1 \Rightarrow -x^4 \leq x^4 \cos(2/x) \leq x^4$ . Since  $\lim_{x \rightarrow 0} (-x^4) = 0$  and  $\lim_{x \rightarrow 0} x^4 = 0$ , we have  
 $\lim_{x \rightarrow 0} [x^4 \cos(2/x)] = 0$  by the Squeeze Theorem.

$$\begin{aligned} 46. \lim_{t \rightarrow 0} \frac{\sin^2 3t}{t^2} &= \lim_{t \rightarrow 0} \left( \frac{\sin 3t}{t} \cdot \frac{\sin 3t}{t} \right) = \lim_{t \rightarrow 0} \frac{\sin 3t}{t} \cdot \lim_{t \rightarrow 0} \frac{\sin 3t}{t} \\ &= \left( \lim_{t \rightarrow 0} \frac{\sin 3t}{t} \right)^2 = \left( 3 \lim_{t \rightarrow 0} \frac{\sin 3t}{3t} \right)^2 = (3 \cdot 1)^2 = 9 \end{aligned}$$