

$$4. \lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 + \sec^2 x}{\cos x} = \frac{1 + 1^2}{1} = 2$$

$$8. \lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{\csc \theta} = \frac{0}{1} = 0. \quad \text{L'Hospital's Rule does not apply.}$$

$$20. \lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec x} = \frac{1 - 1}{1} = 0. \quad \text{L'Hospital's Rule does not apply.}$$

$$26. \lim_{x \rightarrow \infty} x \tan(1/x) = \lim_{x \rightarrow \infty} \frac{\tan(1/x)}{1/x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\sec^2(1/x)(-1/x^2)}{-1/x^2} = \lim_{x \rightarrow \infty} \sec^2(1/x) = 1^2 = 1$$

$$32. y = (\tan 2x)^x \Rightarrow \ln y = x \cdot \ln \tan 2x, \text{ so}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} \ln y &= \lim_{x \rightarrow 0^+} x \cdot \ln \tan 2x = \lim_{x \rightarrow 0^+} \frac{\ln \tan 2x}{1/x} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{(1/\tan 2x)(2 \sec^2 2x)}{-1/x^2} \\ &= \lim_{x \rightarrow 0^+} \frac{-2x^2 \cos 2x}{\sin 2x \cos^2 2x} = \lim_{x \rightarrow 0^+} \frac{2x}{\sin 2x} \cdot \lim_{x \rightarrow 0^+} \frac{-x}{\cos 2x} = 1 \cdot 0 = 0 \Rightarrow \end{aligned}$$

$$\lim_{x \rightarrow 0^+} (\tan 2x)^x = \lim_{x \rightarrow 0^+} e^{\ln y} = e^0 = 1.$$

$$36. y = x^{(\ln 2)/(1 + \ln x)} \Rightarrow \ln y = \frac{\ln 2}{1 + \ln x} \ln x \Rightarrow$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{(\ln 2)(\ln x)}{1 + \ln x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{(\ln 2)(1/x)}{1/x} = \lim_{x \rightarrow \infty} \ln 2 = \ln 2,$$

$$\text{so } \lim_{x \rightarrow \infty} x^{(\ln 2)/(1 + \ln x)} = \lim_{x \rightarrow \infty} e^{\ln y} = e^{\ln 2} = 2.$$

$$48. L = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = \lim_{x \rightarrow 0} \frac{\sin 2x + ax^3 + bx}{x^3} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2 \cos 2x + 3ax^2 + b}{3x^2}. \text{ As } x \rightarrow 0, 3x^2 \rightarrow 0, \text{ and}$$

$(2 \cos 2x + 3ax^2 + b) \rightarrow b + 2$, so the last limit exists only if $b + 2 = 0$, that is, $b = -2$. Thus,

$$\lim_{x \rightarrow 0} \frac{2 \cos 2x + 3ax^2 - 2}{3x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-4 \sin 2x + 6ax}{6x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-8 \cos 2x + 6a}{6} = \frac{6a - 8}{6}, \text{ which is equal to 0 if and only if}$$

$a = \frac{4}{3}$. Hence, $L = 0$ if and only if $b = -2$ and $a = \frac{4}{3}$.