

20. $y = f(x) = \sqrt{x/(x-5)}$ A. $D = \{x \mid x/(x-5) \geq 0\} = (-\infty, 0] \cup (5, \infty)$. B. Intercepts are 0.

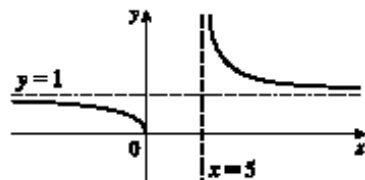
C. No symmetry D. $\lim_{x \rightarrow \pm\infty} \sqrt{\frac{x}{x-5}} = \lim_{x \rightarrow \pm\infty} \sqrt{\frac{1}{1-5/x}} = 1$, so $y = 1$ is a HA. $\lim_{x \rightarrow 5^+} \sqrt{\frac{x}{x-5}} = \infty$, so $x = 5$

is a VA. E. $f'(x) = \frac{1}{2} \left(\frac{x}{x-5} \right)^{-1/2} \cdot \frac{(-5)}{(x-5)^2} = -\frac{5}{2} [x(x-5)^3]^{-1/2} < 0$, so f is decreasing on $(-\infty, 0)$ and $(5, \infty)$.

F. No extreme values G. $f''(x) = \frac{5}{4} [x(x-5)^3]^{-3/2} (x-5)^2 (4x-5) > 0$

for $x > 5$, and $f''(x) < 0$ for $x < 0$, so f is CU on $(5, \infty)$ and

CD on $(-\infty, 0)$. No IP



43. $y = f(x) = xe^{-x^2}$ A. $D = \mathbb{R}$ B. Intercepts are 0 C. $f(-x) = -f(x)$, so the curve is symmetric

about the origin. D. $\lim_{x \rightarrow \pm\infty} xe^{-x^2} = \lim_{x \rightarrow \pm\infty} \frac{x}{e^{x^2}} \stackrel{H}{=} \lim_{x \rightarrow \pm\infty} \frac{1}{2xe^{x^2}} = 0$, so $y = 0$ is a HA.

E. $f'(x) = e^{-x^2} - 2x^2 e^{-x^2} = e^{-x^2} (1 - 2x^2) > 0 \Leftrightarrow x^2 < \frac{1}{2} \Leftrightarrow |x| < \frac{1}{\sqrt{2}}$, so f is increasing on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

and decreasing on $(-\infty, -\frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}}, \infty)$. F. Local maximum value $f(\frac{1}{\sqrt{2}}) = 1/\sqrt{2e}$, local minimum value

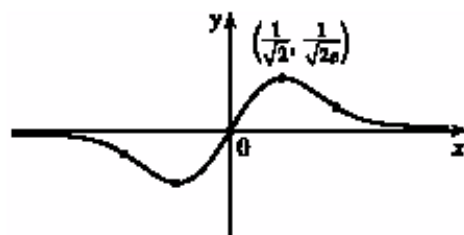
$f(-\frac{1}{\sqrt{2}}) = -1/\sqrt{2e}$ G. $f''(x) = -2xe^{-x^2} (1 - 2x^2) - 4xe^{-x^2} = 2xe^{-x^2} (2x^2 - 3) > 0 \Leftrightarrow$

$x > \sqrt{\frac{3}{2}}$ or $-\sqrt{\frac{3}{2}} < x < 0$, so f is CU on $(\sqrt{\frac{3}{2}}, \infty)$

and $(-\sqrt{\frac{3}{2}}, 0)$ and CD on $(-\infty, -\sqrt{\frac{3}{2}})$ and $(0, \sqrt{\frac{3}{2}})$.

IPs at $(0, 0)$ and $(\pm\sqrt{\frac{3}{2}}, \pm\sqrt{\frac{3}{2}}e^{-3/2})$

H.



4. Let $x > 0$ and let $f(x) = x + 1/x$. We wish to minimize $f(x)$. Now $f'(x) = 1 - \frac{1}{x^2} = \frac{1}{x^2} (x^2 - 1) = \frac{1}{x^2} (x+1)(x-1)$, so the only critical number in $(0, \infty)$ is 1. $f'(x) < 0$ for $0 < x < 1$ and $f'(x) > 0$ for $x > 1$, so f has an absolute minimum at $x = 1$, and $f(1) = 2$.

Or: $f''(x) = 2/x^3 > 0$ for all $x > 0$, so f is concave upward everywhere and the critical point $(1, 2)$ must correspond to a local minimum for f .

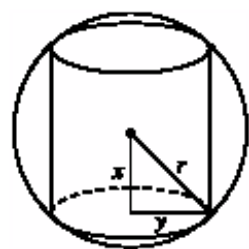
10. Let b be the length of the base of the box and h the height. The volume is $32,000 = b^2 h \Rightarrow h = 32,000/b^2$.

The surface area of the open box is $S = b^2 + 4hb = b^2 + 4(32,000/b^2)b = b^2 + 4(32,000)/b$. So

$S'(b) = 2b - 4(32,000)/b^2 = 2(b^3 - 64,000)/b^2 = 0 \Leftrightarrow b = \sqrt[3]{64,000} = 40$. This gives an absolute minimum since

$S'(b) < 0$ if $0 < b < 40$ and $S'(b) > 0$ if $b > 40$. The box should be $40 \times 40 \times 20$.

17.



The cylinder has volume $V = \pi y^2(2x)$. Also $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2$, so $V(x) = \pi(r^2 - x^2)(2x) = 2\pi(r^2x - x^3)$, where $0 \leq x \leq r$.

$V'(x) = 2\pi(r^2 - 3x^2) = 0 \Rightarrow x = r/\sqrt{3}$. Now $V(0) = V(r) = 0$, so there is a maximum when $x = r/\sqrt{3}$ and

$$V(r/\sqrt{3}) = \pi(r^2 - r^2/3)(2r/\sqrt{3}) = 4\pi r^3/(3\sqrt{3}).$$

24. The volume and surface area of a cone with radius r and height h are given by $V = \frac{1}{3}\pi r^2 h$ and $S = \pi r \sqrt{r^2 + h^2}$.

We'll minimize $A = S^2$ subject to $V = 27$. $V = 27 \Rightarrow \frac{1}{3}\pi r^2 h = 27 \Rightarrow r^2 = \frac{81}{\pi h}$ (1).

$$A = \pi^2 r^2 (r^2 + h^2) = \pi^2 \left(\frac{81}{\pi h} \right) \left(\frac{81}{\pi h} + h^2 \right) = \frac{81^2}{h^2} + 81\pi h, \text{ so } A' = 0 \Rightarrow \frac{-2 \cdot 81^2}{h^3} + 81\pi = 0 \Rightarrow$$

$$81\pi = \frac{2 \cdot 81^2}{h^3} \Rightarrow h^3 = \frac{162}{\pi} \Rightarrow h = \sqrt[3]{\frac{162}{\pi}} = 3\sqrt[3]{\frac{6}{\pi}} \approx 3.722. \text{ From (1), } r^2 = \frac{81}{\pi h} = \frac{81}{\pi \cdot 3\sqrt[3]{6/\pi}} = \frac{27}{\sqrt[3]{6\pi^2}} \Rightarrow$$

$$r = \frac{3\sqrt{3}}{\sqrt[3]{6\pi^2}} \approx 2.632. A'' = 6 \cdot 81^2/h^4 > 0, \text{ so } A \text{ and hence } S \text{ has an absolute minimum at these values of } r \text{ and } h.$$

48. We maximize the cross-sectional area

$$\begin{aligned} A(\theta) &= 10h + 2\left(\frac{1}{2}dh\right) = 10h + dh = 10(10 \sin \theta) + (10 \cos \theta)(10 \sin \theta) \\ &= 100(\sin \theta + \sin \theta \cos \theta), \quad 0 \leq \theta \leq \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} A'(\theta) &= 100(\cos \theta + \cos^2 \theta - \sin^2 \theta) = 100(\cos \theta + 2\cos^2 \theta - 1) \\ &= 100(2\cos \theta - 1)(\cos \theta + 1) = 0 \text{ when } \cos \theta = \frac{1}{2} \Leftrightarrow \theta = \frac{\pi}{3} \quad [\cos \theta \neq -1 \text{ since } 0 \leq \theta \leq \frac{\pi}{2}.] \end{aligned}$$

Now $A(0) = 0$, $A(\frac{\pi}{2}) = 100$ and $A(\frac{\pi}{3}) = 75\sqrt{3} \approx 129.9$, so the maximum occurs when $\theta = \frac{\pi}{3}$.

