

$$7. \int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw = \lim_{t \rightarrow -\infty} \int_t^{-1} \frac{1}{\sqrt{2-w}} dw = \lim_{t \rightarrow -\infty} [-2\sqrt{2-w}]_t^{-1} \quad [u = 2-w, du = -dw]$$

$$= \lim_{t \rightarrow -\infty} [-2\sqrt{3} + 2\sqrt{2-t}] = \infty. \quad \text{Divergent}$$

$$10. \int_{-\infty}^{-1} e^{-2t} dt = \lim_{x \rightarrow -\infty} \int_x^{-1} e^{-2t} dt = \lim_{x \rightarrow -\infty} [-\frac{1}{2}e^{-2t}]_x^{-1} = \lim_{x \rightarrow -\infty} [-\frac{1}{2}e^2 + \frac{1}{2}e^{-2x}] = \infty. \quad \text{Divergent}$$

$$19. \text{Integrate by parts with } u = \ln x, dv = dx/x^2 \Rightarrow du = dx/x, v = -1/x.$$

$$\int_1^{\infty} \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_1^t = \lim_{t \rightarrow \infty} \left(-\frac{\ln t}{t} - \frac{1}{t} + 0 + 1 \right) = -0 - 0 + 0 + 1 = 1$$

$$\text{since } \lim_{t \rightarrow \infty} \frac{\ln t}{t} \stackrel{H}{=} \lim_{t \rightarrow \infty} \frac{1/t}{1} = 0. \quad \text{Convergent}$$

$$27. \text{There is an infinite discontinuity at } x = 1. \quad \int_0^{33} (x-1)^{-1/5} dx = \int_0^1 (x-1)^{-1/5} dx + \int_1^{33} (x-1)^{-1/5} dx.$$

$$\text{Here } \int_0^1 (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^-} \int_0^t (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^-} \left[\frac{5}{4}(x-1)^{4/5} \right]_0^t = \lim_{t \rightarrow 1^-} \left[\frac{5}{4}(t-1)^{4/5} - \frac{5}{4} \right] = -\frac{5}{4}$$

$$\text{and } \int_1^{33} (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^+} \int_t^{33} (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^+} \left[\frac{5}{4}(x-1)^{4/5} \right]_t^{33} = \lim_{t \rightarrow 1^+} \left[\frac{5}{4} \cdot 16 - \frac{5}{4}(t-1)^{4/5} \right] = 20.$$

$$\text{Thus, } \int_0^{33} (x-1)^{-1/5} dx = -\frac{5}{4} + 20 = \frac{75}{4}. \quad \text{Convergent}$$

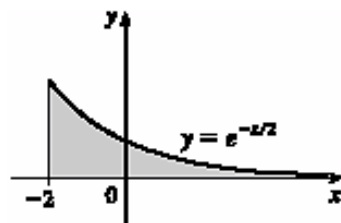
$$29. \text{There is an infinite discontinuity at } x = 0. \quad \int_{-1}^1 \frac{e^x}{e^x - 1} dx = \int_{-1}^0 \frac{e^x}{e^x - 1} dx + \int_0^1 \frac{e^x}{e^x - 1} dx.$$

$$\int_{-1}^0 \frac{e^x}{e^x - 1} dx = \lim_{t \rightarrow 0^-} \int_{-1}^t \frac{e^x}{e^x - 1} dx = \lim_{t \rightarrow 0^-} [\ln|e^x - 1|]_{-1}^t = \lim_{t \rightarrow 0^-} [\ln|e^t - 1| - \ln|e^{-1} - 1|] = -\infty,$$

$$\text{so } \int_{-1}^1 \frac{e^x}{e^x - 1} dx \text{ is divergent. The integral } \int_0^1 \frac{e^x}{e^x - 1} dx \text{ also diverges since}$$

$$\int_0^1 \frac{e^x}{e^x - 1} dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{e^x}{e^x - 1} dx = \lim_{t \rightarrow 0^+} [\ln|e^x - 1|]_t^1 = \lim_{t \rightarrow 0^+} [\ln|e - 1| - \ln|e^t - 1|] = \infty. \quad \text{Divergent}$$

$$34. \quad \text{Area} = \int_{-2}^{\infty} e^{-x/2} dx = -2 \lim_{t \rightarrow \infty} [e^{-x/2}]_{-2}^t = -2 \lim_{t \rightarrow \infty} e^{-t/2} + 2e = 2e$$



46. For $0 \leq x \leq 1$, $e^{-x} \leq 1 \Rightarrow \frac{e^{-x}}{\sqrt{x}} \leq \frac{1}{\sqrt{x}}$.

$$\int_0^1 \frac{1}{\sqrt{x}} dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{1}{\sqrt{x}} dx = \lim_{t \rightarrow 0^+} [2\sqrt{x}]_t^1 = \lim_{t \rightarrow 0^+} (2 - 2\sqrt{t}) = 2 \text{ is convergent.}$$

Therefore, $\int_0^1 \frac{e^{-x}}{\sqrt{x}} dx$ is convergent by the Comparison Theorem.

57. $I = \int_a^\infty \frac{1}{x^2 + 1} dx = \lim_{t \rightarrow \infty} \int_a^t \frac{1}{x^2 + 1} dx = \lim_{t \rightarrow \infty} [\tan^{-1} x]_a^t = \lim_{t \rightarrow \infty} (\tan^{-1} t - \tan^{-1} a) = \frac{\pi}{2} - \tan^{-1} a.$

$$I < 0.001 \Rightarrow \frac{\pi}{2} - \tan^{-1} a < 0.001 \Rightarrow \tan^{-1} a > \frac{\pi}{2} - 0.001 \Rightarrow a > \tan\left(\frac{\pi}{2} - 0.001\right) \approx 1000.$$