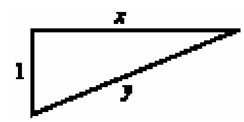
3. Let s denote the side of a square. The square's area A is given by $A = s^2$. Differentiating with respect to t gives us $\frac{dA}{dt} = 2s\,\frac{ds}{dt}$. When $A = 16$, $s = 4$. Substitution 4 for s and 6 for $\frac{ds}{dt}$ gives us $\frac{dA}{dt} = 2(4)(6) = 48\ \text{cm}^2/\text{s}$.

7. $z^2 = x^2 + y^2 \quad \Rightarrow \quad 2z\,\frac{dz}{dt} = 2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt} \quad \Rightarrow \quad \frac{dz}{dt} = \frac{1}{z}\left(x\,\frac{dx}{dt} + y\,\frac{dy}{dt}\right)$. When $x = 5$ and $y = 12$,

$z^2 = 5^2 + 12^2 \quad \Rightarrow \quad z^2 = 169 \quad \Rightarrow \quad z = \pm 13$. For $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 3$, $\frac{dz}{dt} = \frac{1}{\pm 13}\,(5 \cdot 2 + 12 \cdot 3) = \pm\frac{46}{13}$.

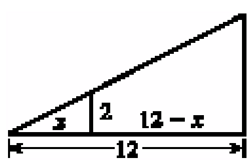
11. (a) Given: a plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. If we let t be time (in hours) and x be the horizontal distance traveled by the plane (in mi), then we are given that $dx/dt = 500$ mi/h.

(b) Unknown: the rate at which the distance from the plane to the station is increasing when it is 2 mi from the station. If we let y be the distance from the plane to the station, then we want to find dy/dt when $y = 2$ mi.

(c) 

(d) By the Pythagorean Theorem, $y^2 = x^2 + 1 \quad \Rightarrow \quad 2y\,(dy/dt) = 2x\,(dx/dt)$.

(e) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{x}{y}(500)$. Since $y^2 = x^2 + 1$, when $y = 2$, $x = \sqrt{3}$, so $\frac{dy}{dt} = \frac{\sqrt{3}}{2}(500) = 250\sqrt{3} \approx 433$ mi/h.

14. 

We are given that $\frac{dx}{dt} = 1.6$ m/s. By similar triangles, $\frac{y}{12} = \frac{2}{x} \quad \Rightarrow \quad y = \frac{24}{x} \quad \Rightarrow$ $\frac{dy}{dt} = -\frac{24}{x^2}\frac{dx}{dt} = -\frac{24}{x^2}(1.6)$. When $x = 8$, $\frac{dy}{dt} = -\frac{24(1.6)}{64} = -0.6$ m/s, so the shadow is decreasing at a rate of 0.6 m/s

20. Let D denote the distance from the origin $(0,0)$ to the point on the curve $y = \sqrt{x}$.

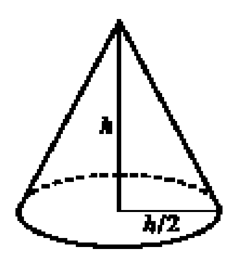
$$D = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (\sqrt{x}\,)^2} = \sqrt{x^2 + x} \quad \Rightarrow \quad \frac{dD}{dt} = \tfrac{1}{2}(x^2 + x)^{-1/2}(2x+1)\,\frac{dx}{dt} = \frac{2x+1}{2\sqrt{x^2+x}}\,\frac{dx}{dt}.$$

With $\frac{dx}{dt} = 3$ when $x = 4$, $\frac{dD}{dt} = \frac{9}{2\sqrt{20}}(3) = \frac{27}{4\sqrt{5}} \approx 3.02$ cm/s.

25. We are given that $\frac{dV}{dt} = 30\ \text{ft}^3/\text{min}$. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12} \quad \Rightarrow$

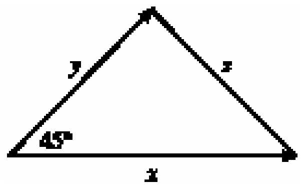
$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \quad \Rightarrow \quad 30 = \frac{\pi h^2}{4}\frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = \frac{120}{\pi h^2}$. When $h = 10$ ft,

$\frac{dh}{dt} = \frac{120}{10^2\pi} = \frac{6}{5\pi} \approx 0.38$ ft/min.



31. With $R_1 = 80$ and $R_2 = 100$, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{80} + \dfrac{1}{100} = \dfrac{180}{8000} = \dfrac{9}{400}$, so $R = \dfrac{400}{9}$. Differentiating $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ with respect to t, we have $-\dfrac{1}{R^2}\dfrac{dR}{dt} = -\dfrac{1}{R_1^2}\dfrac{dR_1}{dt} - \dfrac{1}{R_2^2}\dfrac{dR_2}{dt} \Rightarrow \dfrac{dR}{dt} = R^2\left(\dfrac{1}{R_1^2}\dfrac{dR_1}{dt} + \dfrac{1}{R_2^2}\dfrac{dR_2}{dt}\right)$. When $R_1 = 80$ and $R_2 = 100$, $\dfrac{dR}{dt} = \dfrac{400^2}{9^2}\left[\dfrac{1}{80^2}(0.3) + \dfrac{1}{100^2}(0.2)\right] = \dfrac{107}{810} \approx 0.132\ \Omega/\text{s}$.

36. We are given that $\dfrac{dx}{dt} = 3$ mi/h and $\dfrac{dy}{dt} = 2$ mi/h. By the Law of Cosines,



$z^2 = x^2 + y^2 - 2xy\cos 45^\circ = x^2 + y^2 - \sqrt{2}\,xy \quad \Rightarrow$

$2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} - \sqrt{2}\,x\dfrac{dy}{dt} - \sqrt{2}\,y\dfrac{dx}{dt}$. After 15 minutes $\left[= \frac{1}{4}\text{ h}\right]$,

we have $x = \frac{3}{4}$ and $y = \frac{2}{4} = \frac{1}{2} \quad \Rightarrow \quad z^2 = \left(\frac{3}{4}\right)^2 + \left(\frac{2}{4}\right)^2 - \sqrt{2}\left(\frac{3}{4}\right)\left(\frac{2}{4}\right) \quad \Rightarrow \quad z = \dfrac{\sqrt{13 - 6\sqrt{2}}}{4}$ and

$$\begin{aligned}\frac{dz}{dt} &= \frac{2}{\sqrt{13 - 6\sqrt{2}}}\left[2\left(\tfrac{3}{4}\right)3 + 2\left(\tfrac{1}{2}\right)2 - \sqrt{2}\left(\tfrac{3}{4}\right)2 - \sqrt{2}\left(\tfrac{1}{2}\right)3\right] \\ &= \frac{2}{\sqrt{13 - 6\sqrt{2}}}\,\frac{13 - 6\sqrt{2}}{2} = \sqrt{13 - 6\sqrt{2}} \approx 2.125 \text{ mi/h}.\end{aligned}$$