19. We solve $C = \frac{5}{9}(F - 32)$ for F: $\frac{9}{5}C = F - 32 \Rightarrow F = \frac{9}{5}C + 32$. This gives us a formula for the inverse function, that is, the Fahrenheit temperature F as a function of the Celsius temperature C.

$F \geq -459.67 \Rightarrow \frac{9}{5}C + 32 \geq -459.67 \Rightarrow \frac{9}{5}C \geq -491.67 \Rightarrow C \geq -273.15$, the domain of the inverse function.

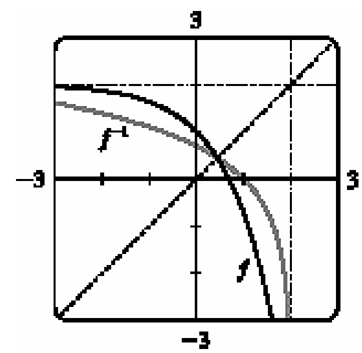
23. $y = f(x) = e^{x^3} \Rightarrow \ln y = x^3 \Rightarrow x = \sqrt[3]{\ln y}$. Interchange x and y: $y = \sqrt[3]{\ln x}$. So $f^{-1}(x) = \sqrt[3]{\ln x}$.

26. $y = f(x) = \dfrac{1 + e^x}{1 - e^x} \Rightarrow y - ye^x = 1 + e^x \Rightarrow y - 1 = ye^x + e^x \Rightarrow y - 1 = e^x(y + 1) \Rightarrow$

$e^x = \dfrac{y - 1}{y + 1} \Rightarrow x = \ln\left(\dfrac{y - 1}{y + 1}\right)$. Interchange x and y: $y = \ln\left(\dfrac{x - 1}{x + 1}\right)$. So $f^{-1}(x) = \ln\left(\dfrac{x - 1}{x + 1}\right)$.

Note that the domain of f^{-1} is $|x| > 1$.

28. $y = f(x) = 2 - e^x \Rightarrow e^x = 2 - y \Rightarrow x = \ln(2 - y)$. Interchange x and y: $y = \ln(2 - x)$. So $f^{-1}(x) = \ln(2 - x)$. From the graph, we see that f and f^{-1} are reflections about the line $y = x$.



35. $f(0) = 1 \Rightarrow f^{-1}(1) = 0$, and $f(x) = x^3 + x + 1 \Rightarrow f'(x) = 3x^2 + 1$ and $f'(0) = 1$. Thus,

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \frac{1}{1} = 1.$$

53. $\ln(1 + x^2) + \frac{1}{2}\ln x - \ln\sin x = \ln(1 + x^2) + \ln x^{1/2} - \ln\sin x = \ln[(1 + x^2)\sqrt{x}] - \ln\sin x = \ln\dfrac{(1 + x^2)\sqrt{x}}{\sin x}$

68. (a) For $f(x) = \ln(2 + \ln x)$, we must have $2 + \ln x > 0 \Rightarrow \ln x > -2 \Rightarrow x > e^{-2}$. Thus, the domain of f is (e^{-2}, ∞).

(b) $y = f(x) = \ln(2 + \ln x) \Rightarrow e^y = 2 + \ln x \Rightarrow \ln x = e^y - 2 \Rightarrow x = e^{e^y - 2}$. Interchange x and y: $y = e^{e^x - 2}$. So $f^{-1}(x) = e^{e^x - 2}$. The domain of f^{-1}, as well as the range of f, is $\mathbb{R}$.

73. $\displaystyle\lim_{x\to\infty}[\ln(1 + x^2) - \ln(1 + x)] = \lim_{x\to\infty}\ln\frac{1 + x^2}{1 + x} = \ln\left(\lim_{x\to\infty}\frac{1 + x^2}{1 + x}\right) = \ln\left(\lim_{x\to\infty}\frac{\frac{1}{x} + x}{\frac{1}{x} + 1}\right) = \infty$, since the limit in parentheses is ∞.