

11. $F(t) = \ln \frac{(2t+1)^3}{(3t-1)^4} = \ln(2t+1)^3 - \ln(3t-1)^4 = 3\ln(2t+1) - 4\ln(3t-1) \Rightarrow$

$$F'(t) = 3 \cdot \frac{1}{2t+1} \cdot 2 - 4 \cdot \frac{1}{3t-1} \cdot 3 = \frac{6}{2t+1} - \frac{12}{3t-1}, \text{ or combined, } \frac{-6(t+3)}{(2t+1)(3t-1)}.$$

13. $f(u) = \frac{\ln u}{1 + \ln(2u)} \Rightarrow$

$$f'(u) = \frac{[1 + \ln(2u)] \cdot \frac{1}{u} - \ln u \cdot \frac{1}{2u} \cdot 2}{[1 + \ln(2u)]^2} = \frac{\frac{1}{u}[1 + \ln(2u) - \ln u]}{[1 + \ln(2u)]^2} = \frac{1 + (\ln 2 + \ln u) - \ln u}{u[1 + \ln(2u)]^2} = \frac{1 + \ln 2}{u[1 + \ln(2u)]^2}$$

14. $y = \ln(x^4 \sin^2 x) = \ln x^4 + \ln(\sin x)^2 = 4 \ln x + 2 \ln \sin x \Rightarrow y' = 4 \cdot \frac{1}{x} + 2 \cdot \frac{1}{\sin x} \cdot \cos x = \frac{4}{x} + 2 \cot x$

39. $y = x \ln x \Rightarrow y' = x(1/x) + (\ln x) \cdot 1 = 1 + \ln x \Rightarrow y'' = 1/x$

44. $f(x) = \ln \ln \ln x \Rightarrow f'(x) = \frac{1}{\ln \ln x} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}.$

$$\text{Dom}(f) = \{x \mid \ln \ln x > 0\} = \{x \mid \ln x > 1\} = \{x \mid x > e\} = (e, \infty).$$

49. $y = x^x \Rightarrow \ln y = \ln x^x \Rightarrow \ln y = x \ln x \Rightarrow y'/y = x(1/x) + (\ln x) \cdot 1 \Rightarrow y' = y(1 + \ln x) \Rightarrow y' = x^x(1 + \ln x)$

57. $y = \ln(x^2 + y^2) \Rightarrow y' = \frac{1}{x^2 + y^2} \frac{d}{dx}(x^2 + y^2) \Rightarrow y' = \frac{2x + 2yy'}{x^2 + y^2} \Rightarrow x^2 y' + y^2 y' - 2yy' = 2x \Rightarrow (x^2 + y^2 - 2y)y' = 2x \Rightarrow y' = \frac{2x}{x^2 + y^2 - 2y}$

61. $y = Ae^{-x} + Bxe^{-x} \Rightarrow y' = A(-e^{-x}) + B[x(-e^{-x}) + e^{-x} \cdot 1]$
 $= -Ae^{-x} + Be^{-x} - Bxe^{-x} = (B - A)e^{-x} - Bxe^{-x}$
 $\Rightarrow y'' = (B - A)(-e^{-x}) - B[x(-e^{-x}) + e^{-x} \cdot 1]$
 $= (A - B)e^{-x} - Be^{-x} + Bxe^{-x} = (A - 2B)e^{-x} + Bxe^{-x},$

so $y'' + 2y' + y = (A - 2B)e^{-x} + Bxe^{-x} + 2[(B - A)e^{-x} - Bxe^{-x}] + Ae^{-x} + Bxe^{-x}$
 $= [(A - 2B) + 2(B - A) + A]e^{-x} + [B - 2B + B]xe^{-x} = 0.$