

$$10. \lim_{x \rightarrow \infty} \frac{\ln \ln x}{x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{1}{x \ln x} = 0$$

$$13. \text{ This limit has the form } \frac{0}{0}. \quad \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$$

$$24. \lim_{x \rightarrow 0^+} \sin x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\csc x} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-\csc x \cot x} = - \lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} \cdot \tan x \right) = - \left(\lim_{x \rightarrow 0^+} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0^+} \tan x \right) \\ = -1 \cdot 0 = 0$$

27. As $x \rightarrow \infty$, $1/x \rightarrow 0$, and $e^{1/x} \rightarrow 1$. So the limit has the form $\infty - \infty$ and we will change the form to a product by factoring out x .

$$\lim_{x \rightarrow \infty} (xe^{1/x} - x) = \lim_{x \rightarrow \infty} x(e^{1/x} - 1) = \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1}{1/x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^{1/x}(-1/x^2)}{-1/x^2} = \lim_{x \rightarrow \infty} e^{1/x} = e^0 = 1$$

$$34. y = \left(1 + \frac{a}{x}\right)^{bx} \Rightarrow \ln y = bx \ln\left(1 + \frac{a}{x}\right), \text{ so}$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{b \ln(1 + a/x)}{1/x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{b \left(\frac{1}{1 + a/x} \right) \left(-\frac{a}{x^2} \right)}{-1/x^2} = \lim_{x \rightarrow \infty} \frac{ab}{1 + a/x} = ab \Rightarrow$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx} = \lim_{x \rightarrow \infty} e^{\ln y} = e^{ab}.$$

$$\begin{aligned}
 43. \quad \lim_{E \rightarrow 0^+} P(E) &= \lim_{E \rightarrow 0^+} \left(\frac{e^E + e^{-E}}{e^E - e^{-E}} - \frac{1}{E} \right) \\
 &= \lim_{E \rightarrow 0^+} \frac{E(e^E + e^{-E}) - 1(e^E - e^{-E})}{(e^E - e^{-E})E} = \lim_{E \rightarrow 0^+} \frac{Ee^E + Ee^{-E} - e^E + e^{-E}}{Ee^E - Ee^{-E}} \quad \left[\text{form is } \frac{0}{0} \right] \\
 &\stackrel{H}{=} \lim_{E \rightarrow 0^+} \frac{Ee^E + e^E \cdot 1 + E(-e^{-E}) + e^{-E} \cdot 1 - e^E + (-e^{-E})}{Ee^E + e^E \cdot 1 - [E(-e^{-E}) + e^{-E} \cdot 1]} \\
 &= \lim_{E \rightarrow 0^+} \frac{Ee^E - Ee^{-E}}{Ee^E + e^E + Ee^{-E} - e^{-E}} = \lim_{E \rightarrow 0^+} \frac{e^E - e^{-E}}{e^E + \frac{e^E}{E} + e^{-E} - \frac{e^{-E}}{E}} \quad [\text{divide by } E] \\
 &= \frac{0}{2+L}, \text{ where } L = \lim_{E \rightarrow 0^+} \frac{e^E - e^{-E}}{E} \quad \left[\text{form is } \frac{0}{0} \right] \\
 &\stackrel{H}{=} \lim_{E \rightarrow 0^+} \frac{e^E + e^{-E}}{1} = \frac{1+1}{1} = 2
 \end{aligned}$$

$$\text{Thus, } \lim_{E \rightarrow 0^+} P(E) = \frac{0}{2+2} = 0.$$