27. $s(t) = 3t^4 + 4t^3 - 6t^2 \quad \Rightarrow \quad s'(t) = 12t^3 + 12t^2 - 12t.\quad s'(t) = 0 \quad \Rightarrow \quad 12t(t^2 + t - 1) \quad \Rightarrow$

$t = 0$ or $t^2 + t - 1 = 0$. Using the quadratic formula to solve the latter equation gives us

$t = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \dfrac{-1 \pm \sqrt{5}}{2} \approx 0.618, -1.618$. The three critical numbers are 0, $\dfrac{-1 \pm \sqrt{5}}{2}$.

36. $f(x) = xe^{2x} \quad \Rightarrow \quad f'(x) = x(2e^{2x}) + e^{2x} = e^{2x}(2x + 1)$. Since e^{2x} is never 0, we have $f'(x) = 0$ only when $2x + 1 = 0 \quad \Leftrightarrow \quad x = -\frac{1}{2}$. So $-\frac{1}{2}$ is the only critical number.

43. $f(t) = t\sqrt{4 - t^2}, \quad [-1, 2]$.

$f'(t) = t \cdot \frac{1}{2}(4 - t^2)^{-1/2}(-2t) + (4 - t^2)^{1/2} \cdot 1 = \dfrac{-t^2}{\sqrt{4 - t^2}} + \sqrt{4 - t^2} = \dfrac{-t^2 + (4 - t^2)}{\sqrt{4 - t^2}} = \dfrac{4 - 2t^2}{\sqrt{4 - t^2}}$.

$f'(t) = 0 \quad \Rightarrow \quad 4 - 2t^2 = 0 \quad \Rightarrow \quad t^2 = 2 \quad \Rightarrow \quad t = \pm\sqrt{2}$, but $t = -\sqrt{2}$ is not in the given interval, $[-1, 2]$.

$f'(t)$ does not exist if $4 - t^2 = 0 \quad \Rightarrow \quad t = \pm 2$, but -2 is not in the given interval. $f(-1) = -\sqrt{3}$, $f(\sqrt{2}) = 2$, and $f(2) = 0$. So $f(\sqrt{2}) = 2$ is the absolute maximum value and $f(-1) = -\sqrt{3}$ is the absolute minimum value.

48. $f(x) = x - \ln x, \; [\frac{1}{2}, 2]. \quad f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}.\; f'(x) = 0 \quad \Rightarrow \quad x = 1$. (Note that 0 is not in the domain of f.)

$f(\frac{1}{2}) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$. So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.