4. (a) $f(x) = \dfrac{x^2}{x^2+3} \quad\Rightarrow\quad f'(x) = \dfrac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2} = \dfrac{6x}{(x^2+3)^2}$. The denominator is positive so the sign of $f'(x)$ is determined by the sign of x. Thus, $f'(x) > 0 \quad\Leftrightarrow\quad x > 0$ and $f'(x) < 0 \quad\Leftrightarrow\quad x < 0$. So f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) f changes from decreasing to increasing at $x = 0$. Thus, $f(0) = 0$ is a local minimum value.

(c) $f''(x) = \dfrac{(x^2+3)^2(6) - 6x \cdot 2(x^2+3)(2x)}{[(x^2+3)^2]^2} = \dfrac{6(x^2+3)\left[x^2+3-4x^2\right]}{(x^2+3)^4} = \dfrac{6(3-3x^2)}{(x^2+3)^3} = \dfrac{-18(x+1)(x-1)}{(x^2+3)^3}$.

$f''(x) > 0 \quad\Leftrightarrow\quad -1 < x < 1$ and $f''(x) < 0 \quad\Leftrightarrow\quad x < -1$ or $x > 1$. Thus, f is CU on $(-1, 1)$ and CD on $(-\infty, -1)$ and $(1, \infty)$. There are IPs at $\left(\pm 1, \frac{1}{4}\right)$.

9. $f(x) = x + \sqrt{1-x} \quad\Rightarrow\quad f'(x) = 1 + \frac{1}{2}(1-x)^{-1/2}(-1) = 1 - \dfrac{1}{2\sqrt{1-x}}$. Note that f is defined for $1 - x \geq 0$; that is, for $x \leq 1$. $f'(x) = 0 \quad\Rightarrow\quad 2\sqrt{1-x} = 1 \quad\Rightarrow\quad \sqrt{1-x} = \frac{1}{2} \quad\Rightarrow\quad 1 - x = \frac{1}{4} \quad\Rightarrow\quad x = \frac{3}{4}$. f' does not exist at $x = 1$, but we can't have a local maximum or minimum at an endpoint.

First Derivative Test: $f'(x) > 0 \quad\Rightarrow\quad x < \frac{3}{4}$ and $f'(x) < 0 \quad\Rightarrow\quad \frac{3}{4} < x < 1$. Since f' changes from positive to negative at $x = \frac{3}{4}$, $f\left(\frac{3}{4}\right) = \frac{5}{4}$ is a local maximum value.

Second Derivative Test: $f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(1-x)^{-3/2}(-1) = -\dfrac{1}{4\left(\sqrt{1-x}\right)^3}$.

$f''\left(\frac{3}{4}\right) = -2 < 0 \quad\Rightarrow\quad f\left(\frac{3}{4}\right) = \frac{5}{4}$ is a local maximum value.

Preference: The First Derivative Test may be slightly easier to apply in this case.

14. (a) f is increasing on the intervals where $f'(x) > 0$, namely, $(2, 4)$ and $(6, 9)$.

(b) f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from positive to negative (at $x = 4$). Similarly, where f' changes from negative to positive, f has a local minimum (at $x = 2$ and at $x = 6$).

(c) When f' is increasing, its derivative f'' is positive and hence, f is CU. This happens on $(1, 3)$, $(5, 7)$, and $(8, 9)$. Similarly, f is CD when f' is decreasing—that is, on $(0, 1)$, $(3, 5)$, and $(7, 8)$.

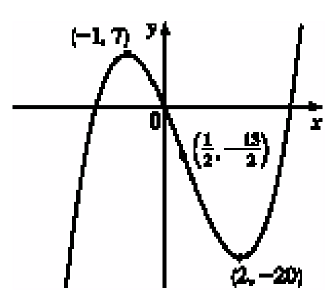
(d) f has IPs at $x = 1$, 3, 5, 7, and 8, since the direction of concavity changes at each of these values.

23. (a) $f(x) = 2x^3 - 3x^2 - 12x \quad\Rightarrow\quad f'(x) = 6x^2 - 6x - 12 = 6\left(x^2 - x - 2\right) = 6(x-2)(x+1)$.
$f'(x) > 0 \quad\Leftrightarrow\quad x < -1$ or $x > 2$ and $f'(x) < 0 \quad\Leftrightarrow\quad -1 < x < 2$.
So f is increasing on $(-\infty, -1)$ and $(2, \infty)$, and f is decreasing on $(-1, 2)$.

(b) Since f changes from increasing to decreasing at $x = -1$, $f(-1) = 7$ is a local maximum value. Since f changes from decreasing to increasing at $x = 2$, $f(2) = -20$ is a local minimum value.

(c) $f''(x) = 6(2x - 1) \quad\Rightarrow\quad f''(x) > 0$ on $\left(\frac{1}{2}, \infty\right)$ and $f''(x) < 0$ on $\left(-\infty, \frac{1}{2}\right)$. So f is CU on $\left(\frac{1}{2}, \infty\right)$ and CD on $\left(-\infty, \frac{1}{2}\right)$. There is a change in concavity at $x = \frac{1}{2}$, and we have an IP at $\left(\frac{1}{2}, -\frac{13}{2}\right)$.

(d)



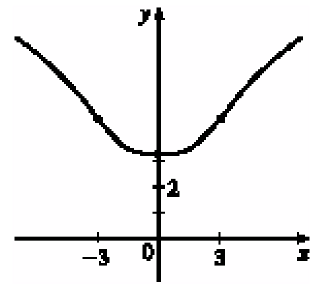
32. (a) $f(x) = \ln(x^4 + 27) \quad \Rightarrow \quad f'(x) = \dfrac{4x^3}{x^4 + 27}$. $f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$, so f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) $f(0) = \ln 27 \approx 3.3$ is a local minimum value.

(c) $$f''(x) = \frac{(x^4+27)(12x^2) - 4x^3(4x^3)}{(x^4+27)^2} = \frac{4x^2\left[3(x^4+27) - 4x^4\right]}{(x^4+27)^2} = \frac{4x^2(81 - x^4)}{(x^4+27)^2} = \frac{-4x^2(x^2+9)(x+3)(x-3)}{(x^4+27)^2}$$

$f''(x) > 0$ if $-3 < x < 0$ and $0 < x < 3$, and $f''(x) < 0$ if $x < -3$ or $x > 3$. Thus, f is CU on $(-3, 0)$ and $(0, 3)$ [hence on $(-3, 3)$] and f is CD on $(-\infty, -3)$ and $(3, \infty)$. There are IPs at $(\pm 3, \ln 108) \approx (\pm 3, 4.68)$.

(d)



39. $f(x) = \ln(1 - \ln x)$ is defined when $x > 0$ (so that $\ln x$ is defined) and $1 - \ln x > 0$ [so that $\ln(1 - \ln x)$ is defined]. The second condition is equivalent to $1 > \ln x \quad \Leftrightarrow \quad x < e$, so f has domain $(0, e)$.

(a) As $x \to 0^+$, $\ln x \to -\infty$, so $1 - \ln x \to \infty$ and $f(x) \to \infty$. As $x \to e^-$, $\ln x \to 1^-$, so $1 - \ln x \to 0^+$ and $f(x) \to -\infty$. Thus, $x = 0$ and $x = e$ are VAs. There is no HA.

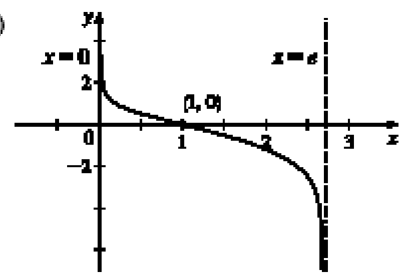
(b) $f'(x) = \dfrac{1}{1 - \ln x}\left(-\dfrac{1}{x}\right) = -\dfrac{1}{x(1 - \ln x)} < 0$ on $(0, e)$. Thus, f is decreasing on its domain, $(0, e)$.

(c) $f'(x) \neq 0$ on $(0, e)$, so f has no local maximum or minimum value.

(d) $$f''(x) = -\frac{-[x(1 - \ln x)]'}{[x(1 - \ln x)]^2} = \frac{x(-1/x) + (1 - \ln x)}{x^2(1 - \ln x)^2} = -\frac{\ln x}{x^2(1 - \ln x)^2}$$

so $f''(x) > 0 \quad \Leftrightarrow \quad \ln x < 0 \quad \Leftrightarrow \quad 0 < x < 1$. Thus, f is CU on $(0, 1)$ and CD on $(1, e)$. There is an IP at $(1, 0)$.

(e)



50. $f(x) = axe^{bx^2} \quad \Rightarrow \quad f'(x) = a\left[xe^{bx^2} \cdot 2bx + e^{bx^2} \cdot 1\right] = ae^{bx^2}\left(2bx^2 + 1\right)$. For $f(2) = 1$ to be a maximum value, we must have $f'(2) = 0$. $f(2) = 1 \quad \Rightarrow \quad 1 = 2ae^{4b}$ and $f'(2) = 0 \quad \Rightarrow \quad 0 = (8b + 1)ae^{4b}$. So $8b + 1 = 0 \;\; [a \neq 0] \quad \Rightarrow$ $b = -\frac{1}{8}$ and now $1 = 2ae^{-1/2} \quad \Rightarrow \quad a = \sqrt{e}/2$.